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A User's Manual For The Sandia One-Dimensional Direct And Inverse Thermal (SODDIT) Code

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Abstract

The SODDIT computer code is a reliable tool for solving a wide variety of one-dimensional transient heat conduction problems. Originally developed in 1972 to predict the ablation of graphite/carbon bodies reentering the earth's atmosphere, it has since been modified by the authors to extend its capabilities well beyond its original scope. This manual describes the SODDIT code in detail and instructs the reader in its use. Several example problems of both direct and inverse heat conduction are presented, and the solutions obtained from SODDIT are compared with the solutions obtained from other sources. Examples of printed output from SODDIT are included.

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Preface

The original Sandia One Dimensional Direct and Inverse Thermal (SODDIT) code was developed in 1972 with its primary emphasis placed upon predicting the ablation of graphite/carbon bodies reentering the earth's atmosphere. Other boundary condition options of the code were available at that time but were not extensively checked. Beginning in 1979, the first author's job assignments became more related to determining heat flux from experimentally measured interior temperatures (the so-called inverse problem of heat conduction). At that point, the code was extensively modified to efficiently solve the inverse heat conduction problem using sensitivity coefficients and the future temperature method of Beck. In 1985, the second author modified SODDIT to include time integration other than the fully implicit method, and also added the energy source capability for both direct and inverse problems. Since the basic elements of the code have existed for a number of years, it has become a reliable and useful tool for solving a wide variety of one-dimensional transient heat conduction problems.

Since the graphite/carbon ablation option has not been used in a number of years, it is presently treated as being inoperable. The reader will find some reference made to ablation problems in the input section because they are a holdover from the original code. If it becomes desirable to again solve ablation problems using SODDIT, it would be relatively easy for the authors to reactivate this option of the code.

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Symbols

English

a	Width of problem domain (Section 5)
A	Area across which energy is diffused
A_c	Cross-sectional area of a one-dimensional fin (Section 5)
A_o	Area of the front-face of the domain, or a constant in the generation-rate function (Section 1), or magnitude of the generation rate per unit volume (Section 5)
A_j	Coefficient in the nodal equation for T_j^{n+1} (3-21)
A_{min}	The smaller of A_i and A_{i+1}, the surface areas of the two materials defining a gap
AREAI (J)†	INPUT: cross sectional area of the J-th element (Block 6-B)
AWALL1	INPUT: radiation absorptivity of the front face (Block 7)
AWALLN	INPUT: radiation absorptivity of the back face (Block 7)
b_n	N-th eigenvalue (Section 5)
B_o	Constant in the generation rate function (Section 1)
B_j	Coefficient in the nodal equation for T_j^{n+1} (3-21)
BP = B′	OUTPUT: dimensionless ablation mass flux
c_r	Reference specific heat value (5-21)
$C^{(j)}$	Capacitance of element j (3-9)
C_o	Coefficient in the energy generation function (Section 1)
C_h	Convection heat-transfer coefficient based on enthalpy difference (1-3)
C_p	Constant pressure specific heat of the material in the problem domain
CH	OUTPUT: if Type 1 boundary condition, interpret as the Stanton number of the boundary fluid, or if Type 3 condition, interpret as the convection heat transfer coefficient h of the boundary
CHO	OUTPUT: heat-transfer Stanton number for zero blowing
CH/CHO	OUTPUT: ratio of CH and CHO
d_i	A component of D_j (3-21)
D_j	Coefficient of the nodal equation for T_j^{n+1} (3-21)
DEPTH	OUTPUT: nodal location below the front face boundary
DHF	INPUT: heat of formation of the j-th material in the computational domain (Block 5)
DTEMPM	INPUT: maximum allowable temperature change of a node in a time step (Block 4)
DTMAX	INPUT: maximum allowable computational time step (Block 4)
DTMIN	INPUT: minimum allowable computational time step (Block 4)
DTPRNT(I)	INPUT: time increment for printing results (Block 3)
DX1(I)	INPUT: thickness of the first element of region in a geometric progression of element sizes (Block 6-A)
DXF(I)	INPUT: thickness of the last element of a region in a geometric progression of element sizes (Block 6-A)
DXI(J)	INPUT: thickness of the j-th element (Block 6-B)
e	Error in the surface energy balance (3-16)
E	Depth of a temperature sensor below the heated surface
EXPN	INPUT: exponent used to generate the cross-sectional area in a domain (Block 4)
$F^{(j)}$	Energy generation sensitivity to temperature for the j-th element (3-10)
$F_{i\to j}$	Geometric configuration factor for thermal radiation exchange between two gap boundaries
$\mathcal{J}_{i\to j}$	Total radiation conductance between gap materials per unit of A_i (1-19)
Fo_e	Element Fourier number (Section 1)

†INPUT and OUTPUT refer to FORTRAN variables, which are either input data to SODDIT or are printed as output by SODDIT

Symbols (continued)

English (continued)

Fo_s	Sensor Fourier number (Section 1)
h	Convective heat-transfer coefficient
h_{cont}	Contact conductance of two gap materials per unit area
HC	OUTPUT: enthalpy of ablation material at the wall temperature
HR	OUTPUT: for Type 1 boundary condition – recovery enthalpy Type 2 boundary condition – wall temperature Type 3 boundary condition – environment temperature
HW	OUTPUT: enthalpy of the gas adjacent to the surface at the surface temperature
i_r	Recovery enthalpy of the free-stream gas
i_w	Enthalpy of the gas at the wall temperature
IBCTA1	INPUT: number of the front-face boundary condition table (Block 7)
IBCTAN	INPUT: number of the back-face boundary condition table (Block 7)
IBCTY1	INPUT: type of front-face boundary condition (Block 7)
IBCTYN	INPUT: type of back-face boundary condition (Block 7)
IFLG	INPUT: flag used in several Blocks to control data input to a given table
IFST	INPUT: index of the first time entry in the front-face boundary condition table of Block 8-I (Block 7)
ISKP	INPUT: increment by which the time index in the sensor table of Block 8-I is to be advanced (Block 7)
ITER	OUTPUT: number of iterations used to satisfy the front face energy balance (Type 1 only)
J_o, J_1	Bessel functions of the first kind of degree 0 and 1, respectively
k	Thermal conductivity of the material in the problem domain
K	In a region of geometrically progressive element sizes, the ratio of adjacent element volumes (1-22)
k_r	Reference thermal conductivity (Section 5)
$K^{(j)}$	Conductance of the j-th element (3-9)
KR(I)	INPUT: integers controlling many options to SODDIT (Block 1)
$\ell^{(j)}$	Length of the j-th element
L	Length of the computational domain
L(T)	Differential operator representing the full heat equation
LUNIT	INPUT: control flag for the length units of the input data (Block1)
$\dot{m}_c''$	Ablation mass flux per unit surface area
MATGEN	INPUT: material number in which energy generation occurs (Block 11)
MATL	OUTPUT: number of material on the left of the current node
MATNAM	INPUT: description of the current material (Block 5)
MATR	OUTPUT: number of the material on the right of the current node
NDTTQ	INPUT: number of times the sensitivity coefficients are computed per problem time step (Block 7)
NER(I)	INPUT: number of elements in the i-th region XR (Block 6-A)
NFT	INPUT: number of future times used to estimate the boundary heat flux (Block 7)
NGAP(J)	INPUT: control flag invoking the GAP option (Block 6-B)
NINV(I)	INPUT: node numbers of sensors (Block 8-I)
NINVN	INPUT: total number of sensors (Block 8-I)
NMAT(J)	INPUT: material number for element-j (Block 6-B)
NMATR(I)	INPUT: i-th material property number (Block 6-A)
NODE	INPUT: node number (Block 6-B)
NTE	Number of total elements
NTN	Number of total nodes (NTE + 1)
Nu	Nusselt number of the gap (based on gap spacing)

Symbols (continued)

English (continued)

P	Pressure of ambient gas, or perimeter of a one-dimensional fin (Section 5)
q''	Heat flux at the surface
$\hat{q}^m$	Value of q_o'' obtained from (1-10) at time m Δt (1-13)
q''_{cond}	Computed surface heat flux (4-13)
q''_{cont}	Heat flux across a gap or interface
q''_{diff}	Surface heat flux due to convection (3-16)
q''_{net}	Net heat flux into a surface (3-16)
q_{new}	Known part of the surface heat flow at time (M + 1) Δt (3-15)
q_{old}	Surface heat flow at time m Δt (3-15)
q''_{rad}	Radiant energy flux into a boundary
$q''_{rad_{in}}$	Radiant input flux absorbed by a surface (3-16)
$q''_{rad_{out}}$	Radiant flux exchange between a surface and its surroundings (3-16)
q''_s	Surface heat flux
Q*	Energy absorbed or liberated during ablation process, per unit mass
$\dot{Q}$	Energy generation rate per unit volume (1-1)
QA, QB, QC, QD, QE	INPUT: constants in the energy-generation-rate function (Block 11)
QCHEM	OUTPUT: heat flow due to phase change, identically zero in nonablating cases
QCOND	OUTPUT: computed heat flux conducted into the material (4-13,14)
QDIFF	OUTPUT: computed convection heat flux (4-1,15)
QNET	OUTPUT: sum of QDIFF, QRIN, and −QROUT (4-16)
QRIN	OUTPUT: computed radiant heat flux absorbed at the boundary (4-2)
QROUT	OUTPUT: computed net radiant flux exchanged between the surface and its environment (4-3)
r	Radial coordinate
r_{eq}	Location of control volume interface such that the element volumes are equally divided
R_i, R_o	Inner and outer radius, respectively, of the computational domain for cylindrical and spherical shells
RADIUS	INPUT: the coordinate location of node 1 (the front face) (Block 4) OUTPUT: the coordinate location of node 1 (the front face)
RE	Smallest of RE_1 and RE_2; relative error in the surface energy balance (3-19)
RE_1	Relative error due to forcing surface energy balance relative error to be small (3-19)
RE_2	Relative error due to forcing the iterated surface temperature changes to be small (3-19)
RHO	INPUT: density of the j-th material (Block 5)
$\dot{s}$ = SDOT	OUTPUT: surface recession rate
S	Squared error (1-9), or OUTPUT: total surface recession distance
SIGMA	OUTPUT: Stéfan-Boltzmann constant
t	Time
Δt	Time-step size
T	Temperature
T*	Approximate solution to L(T) = 0
$T_j(t)$	Time history of the temperature at node j
$\dot{T}_j$	Time derivative of the temperature at node j
T_j^n	Temperature at node j at time t_n
T_r	Partial derivative of T with respect to the radial coordinate, or the reference temperature in (5-21)
T_{rad}	Absolute temperature of the environment for radiant exchange
T_{sg}	Guessed value of the surface temperature

Symbols (continued)

English (continued)

T_w	Wall or surface temperature
T_x	Partial derivative of temperature with respect to the x coordinate
T_o	Fixed boundary temperature (Section 5)
T_∞	Environment temperature for convection heat transfer
T(J)	INPUT: initial temperature of node j (Block 6-B)
TBP	INPUT: dimensionless mass loss rate (Block 10)
TCALC	OUTPUT: calculated value of surface node temperature for Type 1 boundary conditions
TCP	INPUT: tabular heat capacity of the current material (Block 5)
TCH	INPUT: tabulated convection heat-transfer coefficient (Blocks 8-D,9)
TEMIT	INPUT: tabulated emissivity (Block 5)
TEMPERATURE	OUTPUT: computed nodal values of temperature
THETA	INPUT: weighting for time differencing scheme (0. – fully explicit, 0.5 – Crank-Nicolson, 1. – fully implicit)
THR	INPUT: Type 1: tabulated recovery enthalpy Type 2: tabulated surface temperature Type 3: tabulated environment temperature (Blocks 8-D,9)
THW	INPUT: tabulated wall enthalpy of gas adjacent to the front face (Block 10)
TINIT	INPUT: initial temperature of the problem domain (Block 6-A)
TINPUT	OUTPUT: (SUMMARY) a listing of the known nodal (i.e., sensor) temperatures
TITLE(I)	INPUT: heading description (Block 2)
TK	INPUT: tabulated thermal conductivity (Block 5)
TP	INPUT: tabulated gas pressure (Block 10)
TPRNT(I)	INPUT: times when printing frequency of the output should change (Block 3)
TQRAD	INPUT: tabulated radiant flux incoming to the boundary (Blocks 8-D,9)
TRAD1	INPUT: temperature with which node 1 exchanges radiation (Block 7)
TRADN	INPUT: temperature with which node NTN exchanges radiation (Block 7)
TREFF	INPUT: reference temperature for DHF (Block 5)
TSURF	OUTPUT: computed surface temperature
TT	INPUT: tabulated temperature values (Block 5)
TTIME	INPUT: tabulated times for the boundary condition tables (Blocks 8-D, 8-I,9)
$\mathcal{V}^{(j)}$	Volume of the j-th element
VOLI(J)	INPUT: volume of element j in input units (Block 6-B)
x	Spatial coordinate for planar geometry
Δx_{gap}	Width of the gap between adjacent materials
x_i	Location of either of the boundaries, x_o or x_L
x_j	Location of the j-th node
x_{j-}, x_{j+}	Location of the left- and right-hand control volume boundaries that surround node j
x_J	Location of the last node in a region of geometrically progressive control volume sizes
Δx_J	Width of the last volume in a region of geometrically progressive control volume sizes
x_L	Location of the last node in the domain; the back face
x_R	Thickness of a region of geometrically progressive control volume sizes
x_o	Location of the first node in the domain; the front face
Δx_1	Width of the first control volume in a region of geometrically progressive control volume sizes
XR(I)	INPUT: thickness of the i-th subregion (Block 6-A)
XTC(I)	INPUT: desired depths for printing summary temperatures (Block 3)
Y(J,K)	INPUT: temperature of sensor K at time step J (Block 8-I)
Y_j	Known temperature history at the j-th location
Z	Axial length of a cylindrical-shell domain
z_j^{m+i-1}	Step-function sensitivity coefficient (1-12)

Symbols (concluded)

Greek

α	Thermal diffusivity of the domain material, $k/\rho\ C_P$
α_w	Absorptivity of the wall material
β	Coefficient used in (5-21)
$\delta(x - x_j)$	Dirac delta function; cf equation above Equation 3-3
ϵ	Emissivity of the wall material
θ	Weighting coeficient used in the time differencing scheme; cf THETA
$\theta(x,t)$	Dimensionless temperature (5-22)
θ_w	Boundary value of $\theta(x,t)$; cf (5-24)
θ_x	Partial derivative of θ with respect to x
θ_o	Initial value of $\theta(x,t)$; cf (5-24)
λ	Coefficient that selects the method of discretization used in the nodal equations; cf, Table 4
λ_n	Eigenvalues defined in (5-33)
π	3.14159....
ρ	Density of the domain material
σ	Stéfan-Boltzmann constant
$\phi(x)$	Initial temperature distribution in the domain
$\omega(x)$	Weighting used in deriving the nodal equations from the method of weighted residuals (3-3)

Subscripts

$(\ )_o$	Evaluated at the front face of the domain
$(\ )_j$	Location of the j-th node
$(\ )_L$	Evaluated at the back face of the domain

Superscripts

$(\ )^j$	The j-th element
$(\ )^n$	Evaluated at time step n (time = $n\ \Delta t$)
$(\ )^p$	Iteration number (3-17)

A User's Manual For The Sandia One-Dimensional Direct And Inverse Thermal (SODDIT) Code

1. Introduction

An Overview

SODDIT, the Sandia One-Dimensional Direct and Inverse Thermal code, is a FORTRAN-77-compatible† program designed to solve a wide variety of one-dimensional, transient, thermal diffusion problems. It is an extremely versatile code. Its features include thermo-physical properties of the materials that can be entered as tabular functions of temperature; composite materials can comprise the domain of the problem; general derivative boundary conditions, which include thermal radiation, can be modeled; and temperature sensitive heat sources or sinks can be considered. Problems in either planar, cylindrical, or spherical geometries can be modeled, as well as those problems called quasi-one-dimensional, which have an arbitrary area variation requiring tabulated input geometry data. The code also solves the Inverse Heat Conduction Problem, in which interior temperature measurements are used to calculate the surface heat flux and temperature that caused the interior temperature response. It is the intent of this manual to lead the user through the code—from the philosophy behind the code, through the development of the program, to data input and output—so that useful results can be produced with a minimum of effort.

The manual consists of five major sections aside from this Introduction. First, the input data to the code for direct and inverse problems is discussed. It is intended that the user understands the input to the code before the remaining sections are fully understood; doing so allows one to produce useful results while still mastering the remaining sections. Second, the development of the difference equations is presented in some detail. Third, the output from SODDIT is explained. Fourth, several example calculations are presented for both direct and inverse problems, with the intent being to provide a variety of sample problems with which the new user can verify his understanding of the input to and output from SODDIT, and to compare the results of SODDIT to either exact solutions or to output from other programs. Fifth and finally, each of the routines of SODDIT is briefly outlined. In the balance of this section, the statement of the governing principle used to model the engineering problem, and information regarding the geometry, interface conditions, and inverse problem philosophy are presented.

†The only non-standard FORTRAN 77 coding is in the "NAMELIST" statements and some clock routines which are computer specific.

The Governing Equations

The governing equations will be divided into two subsections: one for direct problems and one for inverse problems.

Direct Problems

Direct problems are those in which boundary and initial conditions are specified and the energy equation is solved to predict the temperature field at all interior locations. In the SODDIT code, the general direct problem is stated as follows:

$$\rho C_p\,(T)\frac{\partial T}{\partial t} = \frac{1}{A}\frac{\partial}{\partial x}\left(k(T)A\frac{\partial T}{\partial x}\right) + \dot{Q}(T),\ 0 < x < L \qquad (1\text{-}1)$$

subject to the initial condition

$$T(x,0) = \phi(x), \quad 0 \le x \le L\ . \qquad (1\text{-}2)$$

The source term $\dot{Q}$ in Eq. (1-1) is shown as a function of temperature, but it could also be an explicit function of either time or space or both. The present version of SODDIT allows for either no sources, a uniform source strength throughout the domain, or a temperature-dependent source. The last case assumes that $\dot{Q}$ (energy per unit time per unit volume) is given by the Arrhenius relation, $\dot{Q} = A_o \exp[-B_o/T(x,t)]$, is a quadratic polynomial of the form $\dot{Q} = A_o + B_o T + C_o T^2$, or is an exponential function of position; details are presented in Chapter 2, Block 11. In these instances, the problems are nonlinear; however, they are linearized by a method to be discussed in Section 3. The form of the thermal source is controlled by SUBROUTINE SOURCE, and it is a simple matter to change this subroutine to allow for other source functions. In the present version of SODDIT, only *one* material of a composite layer may have sources; if this limitation becomes critical, it would be relatively straightforward to modify the code since all the source function evaluations are contained in SUBROUTINE SOURCE.

A specification of the boundary couditions requires some definitions. The terms "front-face" and "back-face" boundary conditions will be used to indicate the surfaces x=0 and x=L, respectively. In a large number of problems, most of the heat transfer takes place at one surface or the other and it is convenient to designate this surface as the "front face." The "front face" can have one of three different types of boundary conditions:

Type 1: Convection/radiation flux, in which convection is given by a recovery enthalpy difference times a transfer coefficient C_h. This boundary condition is useful in simulating aerodynamic heating problems in which the heat flux is time dependent. Ablation is also allowed.

Type 2: Specified surface temperature and recession rate as a function of time.

Type 3: Convection/radiation flux, in which convection is given by a temperature difference multiplied by a transfer coefficient h. This boundary condition is useful for simulating time-dependent low-velocity free and forced convection heat transfer. Ablation is not allowed.

The general Type 1 boundary condition without ablation can be written as

$$-k(T)A_o \left.\frac{\partial T}{\partial x}\right|_{x_0} = C_h(t)\,[i_r(t) - i_w(T_w)] + \alpha_{w_o} A_o\, q''_{rad_o}(t) - \epsilon_{w_o}(T)\sigma\, A_o\,[T^4(x_o,t) - T^4_{rad_o}(t)] \qquad (1\text{-}3)$$

where the subscript "o" means the quantity is evaluated at $x = x_0$. All terms indicated as having an explicit time dependence are input in tabular form. The driving potential for convective heating is the difference in recovery enthalpy between the free stream and the wall; i_r is the recovery enthalpy of the gas at the free-stream conditions and i_w is the gas enthalpy evaluated at the wall temperature; note that both enthalpy terms must have the same reference temperature or base. Figure 1 is a schematic of the Type 1 boundary condition. Due to the generality of Eq. (1-3), other types of boundary conditions can be readily simulated. For example, a pure aerodynamic heating input can be simulated by having $q''_{rad} = 0$ and $\epsilon_w = 0$; a known time-dependent flux can be simulated by $C_h = 0$, $\alpha_w = 1.0$, and $\epsilon_w = 0$; or a pure radiation input with grey body reradiation to an environment at T_{rad} can be simulated if $C_h = 0$. Of course, the most general case is a combination of all three heat transfer mechanisms. Since the enthalpy of the gas at the wall depends upon temperature and reradiation depends on wall temperature to the fourth power, Eq. (1-3) is a nonlinear boundary condition. The SODDIT code iterates until this nonlinear surface energy balance is satisfied; a variant of Newton's method is used and convergence generally takes only 2 to 4 iterations. The Type 1 boundary condition is allowed at only the front face; the back face boundary condition must be either Type 2 or Type 3. They are discussed below.

The Type 2 boundary condition is a specified surface temperature as a function of time and can be applied at either the front or back face or both. Mathematically, this condition is

$$T(0,t) = T_0(t) \text{ and/or } T(L,t) = T_L(t)\ . \qquad (1\text{-}4)$$

This boundary condition is implemented in the form of tabulated temperature as a function of time, and can be applied at the front face and/or the back face; linear interpolation is used at intermediate times. In addition, the front face can also have a specified surface recession rate, input as a tabular function of time.

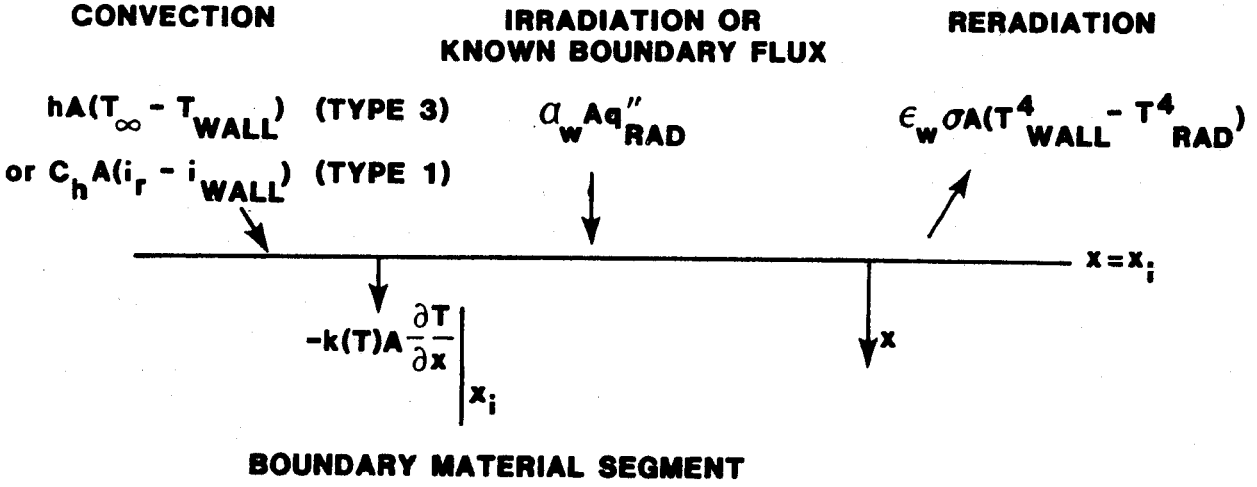


Figure 1. The Elements of the General Derivative or Flux Boundary Condition for Types 1 or 3

The Type 3 boundary condition is quite similar to the Type 1 boundary condition, with the exception being that the convective input is given by a temperature-difference driving potential multiplying a transfer coefficient, h, as given in Eq. (1-5):

$$-k(T)A_i \frac{\partial T}{\partial x}\bigg|_{x_i} = h_i(t)A_i\left[T_{\infty_i}(t) - T(x_i,t)\right] + \alpha_{w_i} A_i q''_{rad_i}(t) - \epsilon_{w_i}(T)\sigma A_i\left[T^4(x_i,t) - T^4_{rad_i}(t)\right] \quad (1\text{-}5)$$

where the subscript "i" refers to either the front or back face of the domain. Although the convection boundary condition is linear provided that h is independent of the wall temperature, the reradiation term is still nonlinear. However, the current version of SODDIT linearizes the radiation term (for the Type 3 boundary condition) so that iteration is not required. The code prints terms that give an indication of the accuracy of this noniterative approach. As in the Type 1 boundary condition case, several other special-case boundary conditions can be simulated with Eq. (1-5) by the proper choice of q''_{rad}, α_w, and and h. Both the front and back face boundaries can have a Type 3 boundary condition.

The primary differences between Type 1 and Type 3 boundary conditions are that the convective flux is given by a different equation and iteration is used to satisfy Type 1 while linearization is used to satisfy Type 3. If the user has a Type 3 boundary condition but feels that the presence of the nonlinear radiation term is justification to iteratively satisfy the surface energy balance, then the code can be "tricked" into satisfying Eq. (1-5) as a Type 1 boundary condition. This is accomplished by specifying a Type 1 boundary condition and the following replacements on the input data:

Type 3 Input	Type 1 Input
T_∞	$i_r = C_p T_\infty$
T_w	$i_w = C_p T_w$
h	$C_h = h/C_p$

where C_p is the (temperature-independent) heat capacity of the gas adjacent to the surface. In fact, this replacement can be accomplished by simply using $C_p = 1$ because i_r, i_w, and C_h are defined such that the heat flux is the same in both cases:

$$q'' = h(T_\infty - T_w) = C_h(i_r - i_w) \; . \quad (1\text{-}6)$$

This allows the Type 3 front-face boundary condition data to be used directly as Type 1 input, provided the thermochemistry data is properly input (see Section 2, Block 10).

Inverse Problems*

Inverse problems attempt to estimate the surface heat flux and/or temperature history at the "active" surface for a material using known (measured) temperature histories at discrete locations inside the material and by knowing the boundary condition at the "inactive" surface. The energy equation and initial conditions as given previously are repeated as

$$\rho C_p(T)\frac{\partial T}{\partial t} = \frac{1}{A}\frac{\partial}{\partial x}\left(k(T)A\frac{\partial T}{\partial x}\right) + \dot{Q}(T) \quad , 0<x<L,\ t>0 \tag{1-1}$$

$$\text{with } T(x,0) = \phi(x), \qquad 0 \le x \le L \ . \tag{1-2}$$

The temperature is known at the J-discrete locations within the material:

$$T(x_j,t) = Y_j(t) \ , \qquad j = 1, 2, \ldots, J \ . \tag{1-7}$$

The "inactive" surface boundary condition is either Type 2 with

$$T(L,t) = T_L(t)$$

or Type 3 where

$$-k(T)A_L\frac{\partial T}{\partial x}\bigg|_L = h_L(t)A_L[T_{\infty_L}(t) - T(L,t)]$$

$$+ \alpha_{w_L} A_L q''_{rad_L}(t) - \epsilon_{w_L}(T) A_L \sigma[T^4(L,t) - T^4_{rad_L}(t)] \ . \tag{1-8}$$

The unknown is the heat flux at the active surface, x=0, since the surface temperature is a by-product of the calculations. The solution for

$$q''_0(t) = -k(T)\frac{\partial T}{\partial x}\bigg|_0$$

is found by minimizing S with respect to $q''_0(t)$, where S is the squared error between the measured (Y) and computed (T) temperature at the J-discrete locations for r-future times.

$$S = \sum_{i=1}^{r}\sum_{j=1}^{J}(Y_j^{m+i-1} - T_j^{m+i-1})^2 \ . \tag{1-9}$$

*Additional details on inverse problems can be found in Beck, Blackwell, and St. Clair [1].

Superscripts refer to the time at which Y_j or T_j should be evaluated [i.e., $t = (m + i - 1)\Delta t$ refers to the time of the "m + i − 1" entry in the table of $Y_j(t)$ with m being the problem's time index]. The index r is the number of future times[1] used in the heat flux computations. A value of $r = 1$ means that $T(x_j,t) = Y_j(t)$ exactly for $J = 1$, whereas a value of $r \ge 2$ means that future temperature data are used to estimate $q''_0(t)$ and is preferred. Using $r > 1$ adds damping to the heat flux errors associated with temperature measurement errors; in affect, it adds stability.

In Eq. (1-9) above, the Y_js are known quantities, but the T_js are unknown in the sense that they are computed from an equivalent direct problem that has an *unknown* boundary flux history at x=0. Thus, the T_js are functions of $q''_0(t)$. The objective is to minimize S with respect to the unknown $q''_0(t)$'s. Thus,

$$\frac{dS}{dq''_0(t)} = \frac{dS}{dq''^m_0} = 0 = \sum_{i=1}^{r}\sum_{j=1}^{J}$$

$$[Y_j^{m+i-1} - T_j^{m+i-1}(q''^m_0)]\left(-\partial T_j^{m+i-1}/\partial q''^m_0\right) \ . \tag{1-10}$$

Let $\hat{q}^m$ be the value of q''_o that is obtained from Eq. (1-10) at time m. Since T_j^{m+i-1} is dependent on $\hat{q}_0^m$, then a two-term Taylor series expansion in heat flux gives

$$T_j^{m+i-1}(\hat{q}_0^m) = T_j^{m+i-1}(\hat{q}_0^{m-1}) + \frac{\partial T_j^{m+i-1}}{\partial \hat{q}_0}\bigg|_{\hat{q}_0^{m-1}} \times$$

$$(\hat{q}_0^m - \hat{q}_0^{m-1}) + \ldots = \overset{*}{T}{}_j^{m+i-1} + \overset{*}{Z}{}_j^{m+i-1} \times$$

$$(\hat{q}_0^m - \hat{q}_0^{m-1}) + \ldots \tag{1-11}$$

where the "*" refers to evaluation at $q'' = \hat{q}_0^{m-1}$ and the Z_j^{m+i-1} are called step function "sensitivity" coefficients and are defined as

$$Z_j^{m+i-1} = \partial T_j^{m+i-1}/\partial q''^m_0 \ . \tag{1-12}$$

Then, substitution of Eq. (1-12) into Eq. (1-10) gives

$$\hat{q}_0^m = \hat{q}_0^{m-1} + \left[\sum_{i=1}^{r}\sum_{j=1}^{J}(Y_j^{m+i-1} - \overset{*}{T}{}_j^{m+i-1})\overset{*}{Z}{}_j^{m+i-1}\right] /$$

$$\sum_{i=1}^{r}\sum_{j=1}^{J}(\overset{*}{Z}{}_j^{m+i-1})^2 \ . \tag{1-13}$$

If the sensitivity coefficients were known, then Eq. (1-13) is a method of finding an estimate of $q_0^{"m}$. To find the sensitivity coefficients, simply differentiate Eq. (1-1) and the appropriate initial/boundary conditions with respect to $q_0^{"}$, the unknown surface flux, giving

$$\rho C_p \frac{\partial Z}{\partial t} = \frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial Z}{\partial x}\right) + \frac{\partial \dot{Q}}{\partial T} Z,\ 0 < x < L,\ t>0 \qquad (1\text{-}14)$$

where, initially, $Z(x,0) = 0$ and with

$$-kA\frac{\partial Z}{\partial x}\bigg|_0 = A(0) \qquad (1\text{-}15)$$

and either $Z(L,t) = 0$, $(1\text{-}16)_1$

if the boundary at $x = L$ has a fixed temperature or

$$-kA\frac{\partial Z}{\partial x}\bigg|_L = -A(L)\left[h_L(t) + 4\epsilon_{w_L}\sigma T^3(L,t)\right]Z(L,t) \qquad (1\text{-}16)_2$$

if it is a derivative boundary. These equations imply that the material properties are constant over r-future time steps so that a series of quasi-constant-property problems are solved. It is noted in Eq. (1-13) that only the sensitivity coefficients at the J – known or measured temperature locations are needed to estimate $q_0^{"}(t)$. Also notice that the system of Eq. (1-14), Eq. (1-15), and Eq. (1-16) is the same sort of problem as the direct diffusion problem, but with altered boundary conditions.

The algorithm for computation of $q_0^{"}(t)$ is outlined as follows:

1. Compute the temperature distribution via the direct problem using r-future times and a guessed value of $q_0^{"} = \hat{q}^{m-1}$ constant for r-future time steps.
2. Compute the Zj values for the r-future times.
3. Using the guessed value of $q_0^{"} = \hat{q}_0^{m-1}$ in Step 1, use Eq. (1-13) to compute the estimate of $\hat{q}_0^m$.
4. With the value of $\hat{q}_0^m$ from Step 3, recompute the temperature field at time t_m and go to Step 1.
5. Proceed to the next time and go to Step 1 so long as the time is less than the maximum time for the problem.

(Note: Even though $q_0^{"}$ is assumed constant over r-future time steps, only that value of $q_m^{"}$ over $t_{m-1} < t < t_m$ is retained.)

The allowable boundary conditions for direct and inverse boundary problems are summarized in Table 1.

Table 1. Allowable Boundary Condition Types for Direct and Inverse Heat Conduction Problems

	Front Face	Back Face
Direct Problems	1, 2, 3	2, 3
Inverse Problems	—	2, 3

Geometrical Concerns

SODDIT is capable of modeling problems in one-dimensional planar, cylindrical, or spherical geomedimensional planar, cylindrical, or spherical geometries as well as those planar geometries that have area as an arbitrary function of position x. These geometries are shown schematically in Figure 2. The area A(x) in the energy equation represents the area through which heat is flowing; that is, A(x) is normal to the heat flux vector at all locations in the material.

In the usual planar geometry, the area is a known constant value and calculations are made on a per-unit-area basis; SODDIT handles this case by internally forcing $A(x) = 1.0$ everywhere in the problem domain. It is also possible to model variable (planar) area by inputting A(x) in tabular form. An example of the variable (planar) area is a tapered fin; however, SODDIT does not handle fin problems per se, unless the lateral side of the fin is insulated or else the convection to or from the fin surface is thought of as a temperature dependent source term [KR(30) = 3]: $\dot{Q} = hT - hT_\infty$ (i.e., $A_0 = -hT_\infty$, $B_0 = h$, $C_0 = 0$). For cylindrical and spherical geometries, the area variation with position can be expressed as

$$A(x) = 2\pi rZ = 2\pi Z(R_i + x) = 2\pi Z(R_0 - x) \quad \text{(cylindrical)} \qquad (1\text{-}17)_1$$

$$A(x) = 4\pi r^2 = 4\pi(R_i + x)^2 = 4\pi(R_0 - x)^2 \quad \text{(spherical)} \qquad (1\text{-}17)_2$$

where R_i and R_0 are defined in Figure 2. These results can be summarized as follows:

$$A(x) \approx r^n \text{ where: } \begin{array}{l}\text{planar geometry, } n = 0\\ \text{cylindrical geometry, } n = 1\\ \text{spherical geometry, } n = 2.\end{array} \qquad (1\text{-}17)_3$$

For any problem with an exponent other than 0, 1, or 2, the area variation must be input to SODDIT in tabular form.

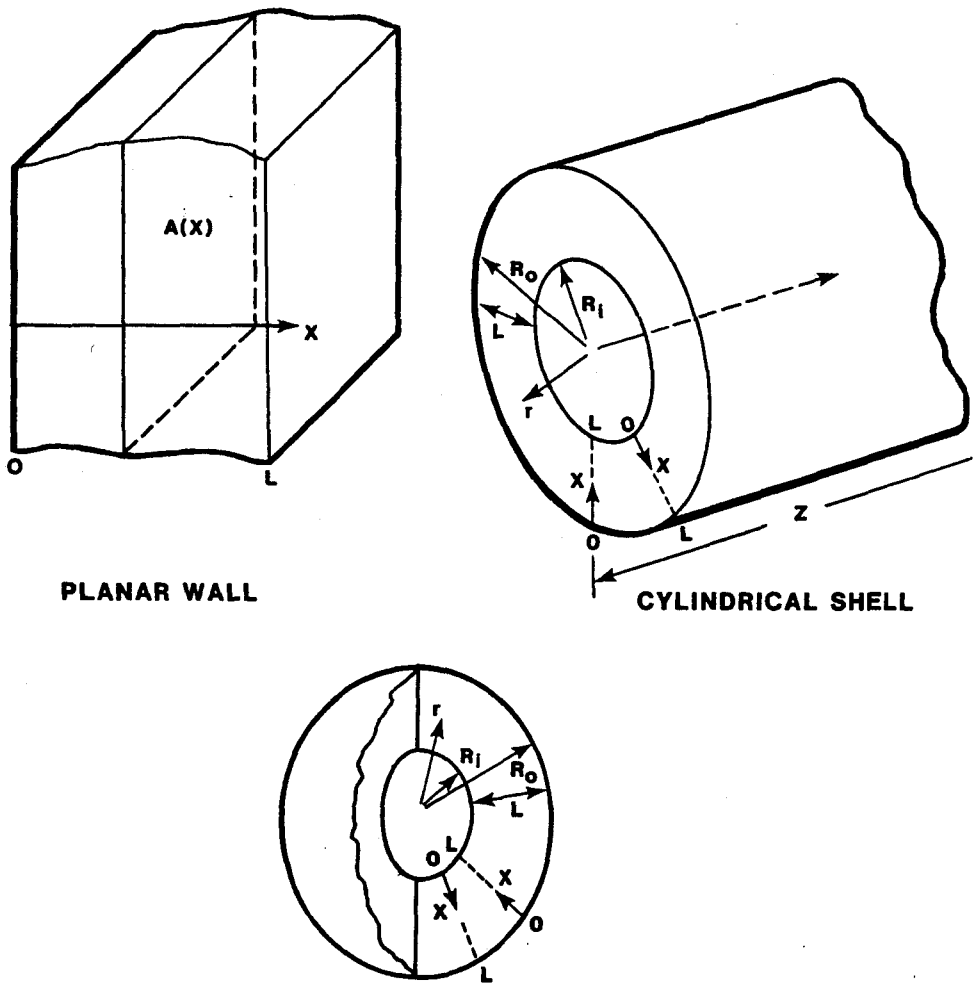


Figure 2. One-Dimensional Geometries Used in SODDIT

When the energy equation is converted from a differential equation to a difference equation, the problem domain will be divided into a series of elements. Each element will have its own value of equivalent radius, area, and volume. For the nonplanar geometries, there is no unique way of choosing radius, area, and volume; consequently, some additional criteria must be specified. In the SODDIT code, the equivalent radius is chosen such that the element *volume* is divided into two *equal* parts. The rationale behind this choice is that the resulting difference equations are symmetric and hence can be solved more efficiently than the nonsymmetric case. Control volume interfaces will be located at r_{eq}. Table 2 summarizes the equations used in SODDIT to calculate the equivalent radius, area, and volume. The absolute value signs are necessary in the volume calculations for the case $r_{j+1} > r_j$. The subscripts j and j+1 on the radius refer to coordinate locations of the element boundaries. Note that Z is the axial length of the domain for cylindrical geometries, and that $r = R - x$ with R the radial location of $x = 0$. $R < 0$ means x ranges from the inner to outer shell radius, and $R > 0$ means x ranges from the outer to inner radius.

Note that x always ranges between 0 and L, the total thickness of the material. For planar slabs, there is no difficulty translating the problem coordinates from, say, an [a,b] interval to the [0,L] interval, where then, $L = b - a$. In cases of cylindrical and spherical geometries, the domain of the problem is spanned by x on [0,L] in one of two ways. Define a radius R, either the inner or outer radius, which locates the point $x = 0$. In this way, x can move from the inner to outer boundary, or vice versa, allowing added flexibility especially needed in inverse heat conduction problems since $x = 0$ *always* defines the boundary whose unknown flux is to be estimated.

Table 2. Geometrical Equations Used in SODDIT. (Note that in cylindrical geometry, SODDIT assumes Z = 1 unit.)

Geometry	r_{eq}	Area	Volume
Planar	$\frac{1}{2}(r_j + r_{j+1})$	1.0	Area $\times$ $\lvert r_j - r_{j+1}\rvert$
Cylindrical	$\left[\frac{1}{2}(r_j^2 + r_{j+1}^2)\right]^{1/2}$	$2\pi r_{eq} Z$	$\pi\lvert r_j^2 - r_{j+1}^2\rvert Z$
Spherical	$\left[\frac{1}{2}(r_j^3 + r_{j+1}^3)\right]^{1/3}$	$4\pi r_{eq}^2$	$\frac{4}{3}\pi\lvert r_j^3 - r_{j+1}^3\rvert$

The user should be aware that in modeling *solid* cylindrical or spherical geometries, the radius R is restricted to positive values because the area at r = 0 is zero and this area is used in the denominator of an equation. More will be said about this in the data input discussion in Section 2.

Interface Conditions

If regions of different materials are in intimate contact, then the interface conditions require that both temperature and heat flux be continuous across the interface. In many situations, the contact is not perfect because of a contact resistance (or conductance). Another situation is a gap of finite width across which radiation and/or convection takes place; any combination posed by this situation is allowed in SODDIT through SUBROUTINE GAP. A schematic of a typical gap is given in Figure 3.

First, consider the case of a contact conductance (resistance) across a gap of *zero width*. The heat transfer rate across such a gap is given by

$$q_{cont} = h_{cont} A_i (T_i - T_{i+1}) = h_{cont} A_{i+1} (T_i - T_{i+1}) \quad \textbf{(1-18)}$$

where $A_i = A_{i+1}$ and $h_{cont}A$ is the reciprocal of the contact resistance between the two surfaces.

For a gap of finite width, the two surfaces are allowed to exchange heat by radiation according to the grey body relationship

$$q_{rad} = A_i \mathcal{J}_{(i)\to(i+1)} \sigma(T_i^4 - T_{i+1}^4)$$

$$= A_{i+1} \mathcal{J}_{(i+1)\to(i)} \sigma(T_i^4 - T_{i+1}^4) \quad \textbf{(1-19)}_1$$

where

$$A_i \mathcal{J}_{i\to j} = \frac{1}{\frac{1-\epsilon_i}{A_i\epsilon_i} + \frac{1}{A_i F_{i\to j}} + \frac{1-\epsilon_j}{A_j\epsilon_j}} . \quad \textbf{(1-19)}_2$$

Note that ϵ_i and A_i are emittance and surface area of surface i and $F_{i\to j}$ is the geometric configuration factor for black and diffuse surfaces. If the analysis is restricted to concentric geometries such that all the radiation leaving the smaller surface will strike the larger surface ($F_{i\to j}$ = 1.0), then

$$A_i F_{i\to j} = A_j F_{j\to i} = \text{MIN}(A_i, A_j) \quad \textbf{(1-20)}$$

where MIN(a,b) is the FORTRAN minimum operator giving the smaller of a and b.

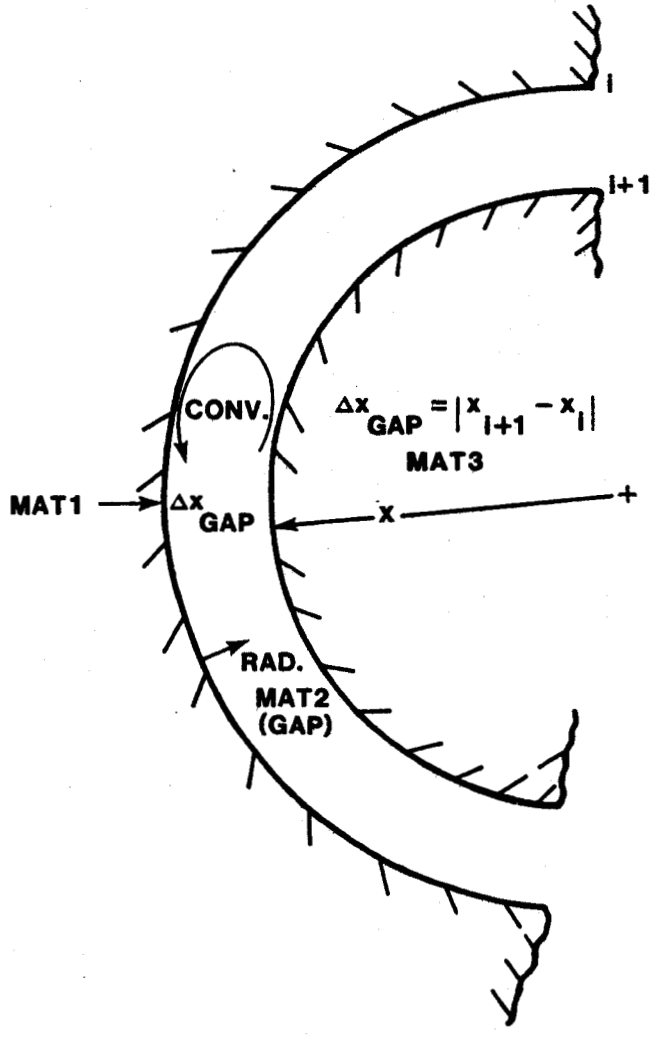


Figure 3. Geometry for a Typical Gap Within the Problem Domain

Convection across the gap is assumed to be of the form

$$q_{conv} = Nu\left(\frac{k}{\Delta x_{gap}}\right) MIN(A_i, A_{i+1}) (T_i - T_{i+1}) \quad (1\text{-}21)$$

where the gap Nusselt number is defined by

$$Nu = \frac{h\Delta x_{gap}}{k}$$

where k is the thermal conductivity of the gap medium (gas or liquid), Δx_{gap} is the gap spacing, and h is the gap convective heat-transfer coefficient. It is assumed that h is based on the minimum area $A_{min} = MIN(A_i, A_{i+1})$; hence, the appearance of $MIN(A_i, A_{i+1})$ in Eq. (1-21). If h were defined using the maximum area, then it would have to be corrected by the appropriate area ratio.

In summary, the "material" properties that one must specify for a gap include Nu(T), k(T), and $h_{cont}(T)$. The radiation properties of surfaces i and i+1 must also be included.

Automatic Grid Generation Using Geometric Progression

To simplify the input to the SODDIT code, an automatic node (or element) generation routine based upon the geometric progression has been developed; details of the input procedure will be described in Block 6-A of Section 2. Figure 4 is a sketch showing four elements, five nodes, and the associated nodal coordinates (x_j) and element size (Δx_j); in this example, each element is 1.5 times as large as the previous element. The grid generation in SODDIT is based upon keeping a fixed ratio between the thickness of adjacent elements; this geometric progression is defined by

$$K = \frac{\Delta x_{j+1}}{\Delta x_j} \quad , j = 1, 2, ---, J-1 \; . \quad (1\text{-}22)$$

The nodal coordinate for K constant is given by

$$x_{j+1} = \Delta x_1(K^j - 1)/(K - 1) \quad , j = 1, 2, ---, J \; . \quad (1\text{-}23)$$

If Eq. (1-23) is evaluated at the last element j = J, the total thickness x_R of the region is given by

$$x_{J+1} = x_R = \Delta x_1(K^J - 1)/(K-1) \quad (1\text{-}24)$$

where $x_1 = 0.0$. At first glance, it might appear desirable to specify the element-thickness-ratio parameter K; however, experience indicates that it is difficult to develop intuition about how to choose K. Instead, two other options that are easier to use are available in SODDIT:

Case A ($J, \Delta x_1$, and x_R): Given the total number of elements J, the thickness of the first element Δx_1, and the total thickness of the region x_R, Eq. (1-24) can be (iteratively) solved for the element-thickness ratio K. Once K is known, all other node and element parameters can be calculated.

A suboption of Case A is to have equal thickness elements in this region. This is accomplished by having K = 1; the logic of the code is such that only J and x_R are required input with $\Delta x_1 = \Delta x_J = x_R/J$.

Case B (Δx_1, x_R, and Δx_J): In Case A above, there is control over the number of elements J but the thickness of the final element Δx_J is determined by the code. In some instances, it may be desirable to specify Δx_J instead of J; for this option, Eq. (1-23) can be evaluated at j = J − 1 to yield

$$x_J = \Delta x_1(K^{J-1} - 1)/(K - 1) \; . \quad (1\text{-}25)$$

Taking the difference between Eqs. (1-24) and (1-25),

$$\Delta x_J = x_{J+1} - x_J = \Delta x_1 K^{J-1} \quad (1\text{-}26)$$

or

$$K = (\Delta x_J/\Delta x_1)^{1/(J-1)} \; . \quad (1\text{-}27)$$

Eliminating K from Eq. (1-24) by means of Eq. (1-27), one obtains

$$x_R = \Delta x_1 \frac{(\Delta x_J/\Delta x_1)^{J/(J-1)} - 1}{(\Delta x_J/\Delta x_1)^{1/(J-1)} - 1} \; . \quad (1\text{-}28)$$

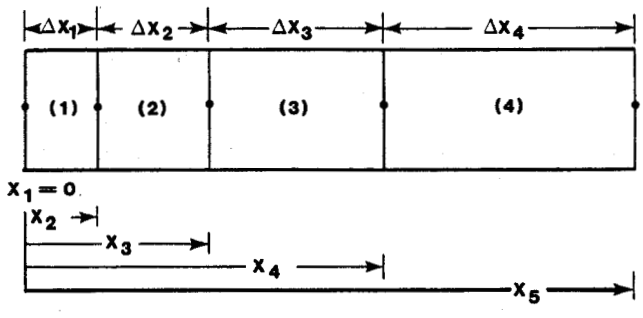


Figure 4. Schematic of Element Generation Using Geometric Progression (J = 4 Elements, K = 1.5)

Given Δx_1, x_R, and Δx_J, Eq. (1-28) is iteratively solved for J. In general, this solution for J will not be integer; the procedure used in SODDIT is to take the integer part of J (round down). Then, knowing J, Δx_1, and x_R, Eq. (1-24) is now solved for K. The resulting value of x_J will be slightly different from the input value in order to have J be an integer.

In order to produce equal thickness elements with this option, only Δx_1 and x_R have to be specified. In this case, $J = INT(x_R/\Delta x_1 + 0.5)$ where INT is the FORTRAN integer operator; next, Δx_1 is recalculated from $\Delta x_1 = x_R/FLOAT(J)$; this recalculation of Δx_1 insures that J, Δx_1, and x_R are all consistent and that $\Delta x_1 = \Delta x_J$.

Philosophy on Solution of Inverse Problems

Although the purely mathematical aspects of the solution of inverse problems have been well established (see Beck, Blackwell, and St. Clair[1] for details), the adjustments necessary for solving real world inverse problems are something that one tends to learn by experience. The purpose of this section is to propose some ideas that have been found to be beneficial in solving inverse problems. A typical example is the problem of using a calorimeter to measure heat fluxes.

The first step in determining the heat flux from interior temperature measurements is the design of the calorimeter. If one is asked to reduce temperature-time data without any input into the calorimeter design, it may be too late to recover meaningful data. Because an interior temperature sensor is both damped and lagged in its response to changes in surface conditions, it may be difficult to recover high-frequency information about the heat flux variation. This implies that the temperature sensor be located as close to the surface as possible. Often, surface-mounted sensors do not give accurate, undisturbed readings of the surface temperature, particularly with thermocouples. An appropriate dimensionless quantity of importance for inverse problems is

$$Fo_s = \alpha\Delta t/E^2$$

where α is the thermal diffusivity, Δt is the data sample rate, and E is the sensor depth below the heated surface. As this parameter, the sensor Fourier number, becomes small, errors in the experimental temperature measurements become increasingly more important. Eventually, the inverse calculation procedure in the SODDIT code will become unstable if this parameter becomes *too small*. Contrast this with the numerical solution of direct heat conduction problems; large values of the element Fourier number

$$Fo_e = \alpha\Delta t/\Delta x^2$$

can cause numerical oscillations in the computed temperature field. Δx is the grid size. Based on computational experiments, recommended computational guidelines for Fo_s are given in Table 3. This information should be interpreted as being a useful starting point, but the user is encouraged to perform his own computational experiments.

Table 3. Recommended Computational Guidelines for Sensor Fourier Number in Terms of Number of Future Times (r)

r	Fo_s
1	1.48
2	0.15
3	0.067
4	0.037

An approach to designing a calorimeter is to *estimate* the heat flux that the calorimeter is expected to see, solve for the temperature response at the sensor location using the direct option of SODDIT, sample the computed temperature-time data at the same rate as the application, perturb the computed temperature data to simulate expected sensor noise, and then run the inverse option of SODDIT. This simulation of sensor noise is a critical step because some inverse methods will work with errorless data but will not work with real data that contains errors. The parameters that one can vary in the design process are sensor depth, calorimeter material, and data sample rate. If the simulated temperature data with errors produces significant oscillations in the computed heat flux, the calorimeter parameters can be adjusted accordingly. For any calorimeter design, there will be a maximum-allowable data sampling rate for which the calculation procedure remains stable. Any further increase in the sampling rate will produce significant oscillations in the calculations. This, in effect, limits the highest frequency component that can be resolved.

Material properties are often taken for granted when using the inverse approach to determine heat flux as a function of time. However, accuracy of

the computed heat flux will at best be equal to the accuracy of the thermal conductivity because of Fourier's law. In reality, the accuracy of the heat flux will be less than the accuracy of the property data because of errors in the temperature measurements. This simply points out that the calorimeter material should have well-characterized thermal properties.

Another material-property-related problem is that some steels go through a magnetic-to-nonmagnetic change (Curie point). The thermal property change associated with passing the Curie point can cause stability problems, as evidenced by the work reported in Gregory, Mata, and Keltner.[2]

Although most digital data aquisition systems sample at a constant rate, there are practical situations in which a nonconstant sample rate might be desirable. In the work of Reference 2, a periodic resistance check was made of thermocouples placed in a fuel-fire environment in order to detect high-temperature anomalies. While the basic sample rate was a constant, every n-th temperature scan was missing. This abrupt change in sample rate caused unexplained stability problems. The solution was to smooth the data and interpolate at the times that the resistance measurements were made.

If sufficient time is taken for the design of the calorimeter, there is a reasonable chance that meaningful information can be obtained from the inverse option of SODDIT.

2. Input File Construction

General Information

The input data for SODDIT is divided into eleven categories of READ-statements or blocks. Each may consist of only a single record or many records of data. A record is defined as a line (or 80 characters of information) and can be thought of as a card (a holdover from the days of computer cards). The various blocks of data are:

1. Control flags
2. Title or heading characters
3. Print intervals and print times for displaying the output data
4. General problem constants
5. Material property data (ρ, C_p, k, ϵ)
6. Initial temperature, nodal and element data
7. Boundary condition constants
8. Boundary condition time table for front face (x = 0)
9. Boundary condition time table for back face (x = L)
10. Enthalpy of boundary layer gas as a function of pressure and temperature, ablation temperature, and heat of ablation
11. Energy generation rate constants

Much of the input information is the same for direct and inverse problems. Options in Blocks 6 and 8 are available. Figure 5 is a flow chart for input showing the options.

NOTE: All integer fields must be right-justified in the format field. Any input quantity that is 0 (integer) or 0.0 can be left blank.

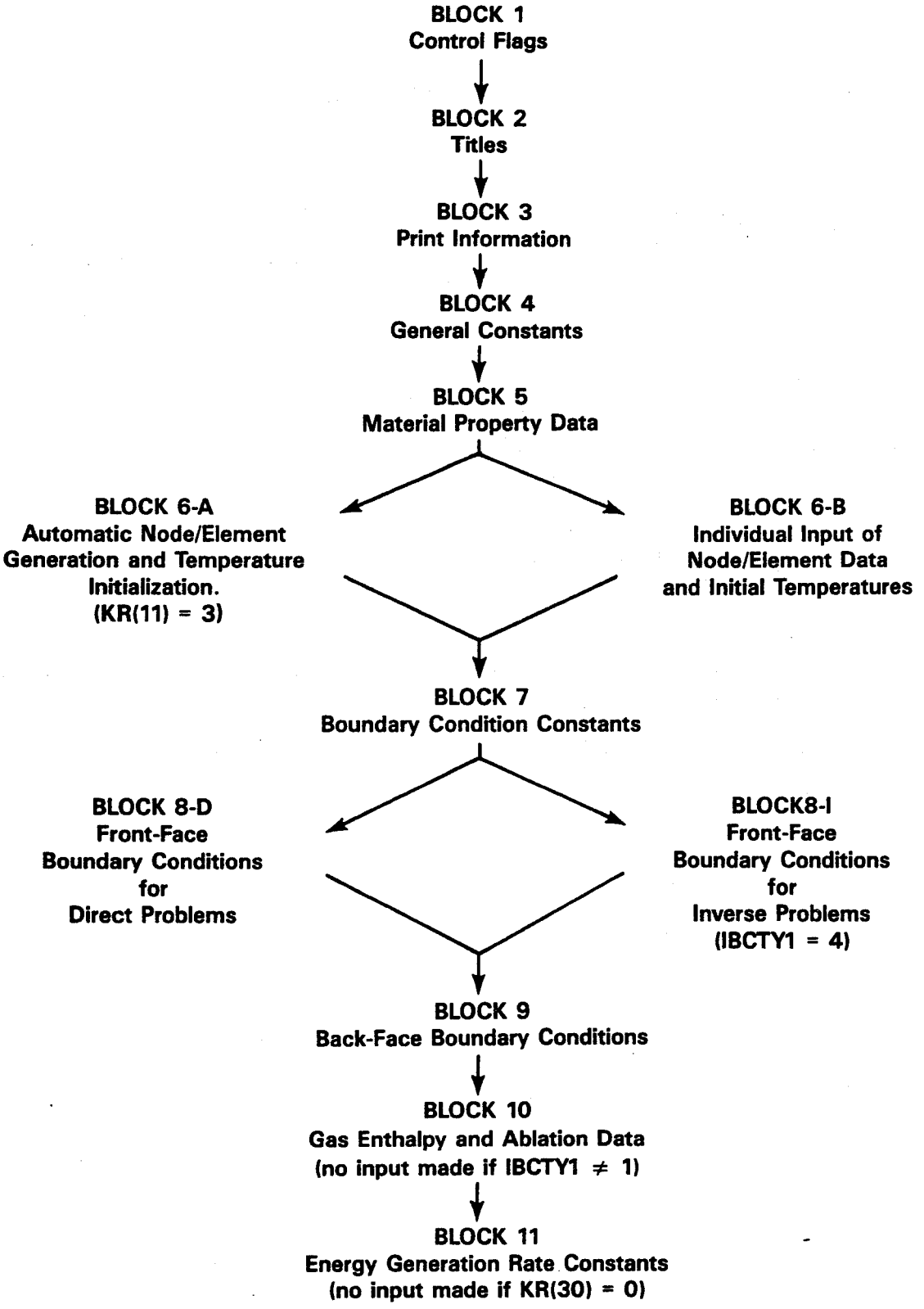


Figure 5. Flowpath for Data Input to SODDIT

Input Data File Construction

BLOCK 1: Control Flags

(1 record)

KR(I), I=1, 80
FORMAT(80I1)

There are 80 flags available for control of various program features. They all appear on one record. As of this date, only 23 flags are used: KR(10) through KR(27) plus KR(30) through KR(33) and KR(40). Unless the flag is specifically mentioned below, it can be left blank on input. The flags presently used are

KR(10) = 0 LUNIT = 1.: all *geometrical* input data have consistent units.

≠ 0 LUNIT = 12.: all *geometrical* input data are in inches and are converted to feet internally. Remaining input data must be in English units, with the exception of gas enthalpy data, Block 10.

KR(11) = 0 User inputs data for node number, material number, element width, initial temperature, element area, element volume, and gap parameter for each element.

= 1 Can be used only when more than one problem is to be solved in a given execution of SODDIT. If KR(11) = 1, then do NOT enter new element/node data, but use the values from the *prior* problem.

= 3 Automatically generate the node and element configuration using SUBROUTINE ANG. (valid only for uniform initial temperature)

KR(12) = 0 Nodal equations derived using finite control volume method but capacitance matrix has been diagonalized (lumped capacitance).

= 1 Nodal equations derived using finite control volume method.

= 2 Nodal equations derived using finite element method.

} See Section 3

KR(13) = 0 User supplies times for printing the output data (Block 3).

≠ 0 Print output data at every time step (useful for debugging). This produces *large* quantities of output.

KR(14) = 0 The quantity TRAD, the temperature to which energy is radiated from a boundary, is a constant, independent of time. This applies to *both* boundaries if both are Type 3 boundaries.

≠ 0 TRAD is a function of time and is set equal to TINF, the ambient temperature for convection heat loss. Applicable only for Type 3 (flux) boundary conditions. Again, this applies to both boundaries if both are Type 3 boundaries.

KR(15) ≠ 0 Debugging information from SUBROUTINE INV, the inverse heat conduction solver, is printed.

KR(16) ≠ 0 Supplementary program output is written on unit TAPE10 for subsequent plotting.

KR(17) $\neq$ 0 Debugging information from SUBROUTINE GETDT, the automatic time-step adjusting routine, is printed.

KR(18) $\neq$ 0 Debugging information from SUBROUTINE GAP, the routine handling heat transfer across gap element, is printed.

KR(19) = 0 Input temperatures in boundary condition tables are in consistent units, do not convert.

= 1 Convert from F to R.

= 2 Convert from C to K.

= 3 Convert from F to K. (Generally used with inverse problems.)

= 4 Convert from C to R.

= 5 Convert from F to C.

= 6 Convert from C to F.

KR(20) = 1-9 An integer such that nodes ISURF+KR(20) and NTN-KR(20) are monitored to control the time step so that their temperatures do not exceed a certain amount, specified at a later point during input. (ISURF and NTN are node numbers of front and back face nodes, ISURF = 1 for non-ablating problems.) This gives the user a means of controlling time step when properties are strongly temperature dependent.

KR(21) $\neq$ 0 Print the tri-diagonal coefficient matrix, useful for debugging.

KR(22) $\neq$ 0 Print the output from SUBROUTINE GETTAB, a routine used to compute the ablation temperature of the material, used for ablation problems only.

KR(23) $\neq$ 0 Print the debug information for gas enthalpy data, used for Type 1 boundary conditions only.

KR(24) = 0 Print summary temperature histories at the material interfaces as well as front and back faces.

$\neq$ 0 Print summary temperature histories at selected thermocouple locations, supplied later in input (Block 3).

KR(25),KR(26) $\neq$ 0 Temperature histories for inverse problems are printed every KR(25)*10 + KR(26) input time steps; convenient for controlling quantity of output.

KR(27) $\neq$ 0 Debugging information from subroutine SRFBNA, the nonablating Type 1 boundary condition solver, is printed.

KR(28) = 0 Thermochemistry data converted to R, Btu/lbm.

≠ 0 Thermochemistry data converted to K, J/g.

KR(30) = 0 No thermal sources are present in the problem.

= 1 A uniform (constant) thermal source is present within the domain of the problem characteristic, supplied in Block 11.

= 2 A temperature-dependent thermal source is present within the domain of the problem of the form:

$\dot{Q} = A\exp(-B/T)$.

The constants A and B are supplied in Block 11 with C = D = E = 0.0 (or blank).

= 3 A temperature-dependent thermal source is present in the domain of the problem of the form:

$\dot{Q} = A + BT + CT^2$

The constants A, B, and C are supplied in Block 11 with D = E = 0.0 (or blank).

= 4 A position- and time-dependent source of the form $\dot{Q} = -I(t)\ (1-R)\ [\exp(-\mu x_{J+1}) - \exp(-\mu x_J)]/\Delta x_J$ where A, B, and C are used to specify I(t) within SUBROUTINE SOURCE, $D=\mu$, and E=R. Physically, this source term represents the indepth absorption of a flux of intensity I(t) incident on a material of reflectance R and absorptance μ. The intensity variation with time I(t) will have to be specified by altering the FORTRAN coding in SUBROUTINE SOURCE. Again, the constants are supplied in Block 11.

KR(31) ≠ 0 The heat of ablation option will be used and both ablation temperature and heat of ablation are required input (Block 10). Otherwise, ignore.

KR(32) = 0 For option 1 ablating surface energy balance, the heat-transfer coefficient is *not* corrected for blowing.

≠ 0 For option 1 ablating surface energy balance, the heat-transfer coefficient is corrected for blowing.

KR(33) = 0 For option 1 surface energy balance, the heat transfer coefficient is *not* corrected for hot wall effects.

≠ 0 For option 1 surface energy balance, the heat transfer coefficient is corrected for hot wall effects.

KR(40) ≠ 0 Print debug information from SUBROUTINES SRFBAB and SRFBNA.

Block 2: Title Records

(3 records)

TITLE(I), I = 1,60
FORMAT (20A4)

Three records must be supplied containing any alpha-numeric information and will be used as output headings. Any or all records may be blank.

Block 3: Print Times, Print Time Increments, and Thermocouple Depths

(3 records)

record 1: TPRNT(I), I = 1, N; N $\leq$ 8; FORMAT (8F10.0), consistent units
record 2: DTPRNT(I), I = 1, N-1; FORMAT (7F10.0), consistent units
record 3: XTC(I), I = 1, M; M $\leq$ 8; FORMAT (8F10.0), consistent units or KR(10) $\neq$ 0. NOTE: Do NOT enter data for record 3 if KR(24) = 0! The record is then omitted from the data file.

TPRNT(I) are used to denote times when the printing frequency of the output should change and DTPRNT(I) are the time increments for printing the output between TPRNT(I) and TPRNT(I+1); of course, DTPRNT should be an integer fraction of the time between consecutive values of TPRNT to insure an integer number of print times. DTPRNT can be input as zero in both direct and inverse problems. If so, only the summary results are printed; refer to Section 4.

Note: the units of TPRNT and DTPRNT must be consistent with the time units of the thermal conductivity of the material and the Stéfan-Boltzmann constant. The number of print times, N, is automatically counted by the program.

The initial problem time is taken to be TPRNT(1) and the final time is TPRNT(N). All problems require at least two values of TPRNT. If KR(13) $\neq$ 0, the temperature profile is printed at every computational time step, overriding the data of record 2; this is useful for obtaining "debug" information. For direct problems (IBCTY $\neq$ 4, not an inverse problem), the temperature print frequency will always be given by DTPRNT. However, in *inverse* problems, DTPRNT must be the difference between TPRNT(I+1) and TPRNT(I), yielding a maximum of seven different print frequencies. DTPRNT = 0. is acceptable, however. Should more than seven values be desired for inverse problems, use KR(25) and KR(26) as described in Block 1.

XTC(I) are the desired depths (not necessarily node locations) for printing summary temperatures; note that these depths are measured from the x=0 surface. SUBROUTINE SUMRY prints these data, but SUBROUTINE OUTPUT actually computes the desired temperatures. If KR(24) = 0, no data are entered for XTC since the program automatically uses the front and back surface nodes and material interface nodes by default in this case. For KR(24) $\neq$ 0, SUBROUTINE OUTPUT linearly interpolates temperature versus position to evaluate the desired temperatures.

Block 4: General Problem Constants

(1 record)

record 1: DTMIN, DTMAX, THETA, DTEMPM, SIGMA, RADIUS, EXPN, FRACT1
FORMAT (4F10.0, E10.4, 3F10.0)

Variable	Description	Default Value	Record Columns
DTMIN[1]	Minimum allowable computational time step, in consistent units, for computing results. It should be left blank for inverse problems, but not for direct problems. Appropriate value depends on problem Fourier number.	0.	1–10
DTMAX[1]	Maximum allowable computational time step, in consistent units, for computing results. It should be left blank for inverse problems, but not for direct problems. Appropriate value depends on problem Fourier number.	0.	11–20
THETA[2]	Weighting factor between 0. and 1. for the time differencing scheme. Fully implicit $\theta = 1.$ and Crank-Nicolson method $\theta = 0.5$ are recommended. For *all* problems involving option 1 surface energy balance, *always* use $\theta = 1.$	1.	21–30
DTEMPM[1]	Maximum allowable temperature change of selected nodes [cf, KR(20)] during one time step (in consistent units).	25.	31–40
SIGMA[3]	Stéfan-Boltzmann constant in consistent units. NOTE: the default value has units of BTU/s-ft^2-R^4.	$4.76111\text{E-}13 = \frac{1.714\text{E-}9}{3600.}$	41–50
RADIUS	Radius of the boundary x = 0 (cf, Figure 2) in units consistent with the geometric (i.e., nodal/element length, area, and volume data) input data. KR(10) $\neq$ 0 will convert RADIUS from inches to feet along with the other geometric input. RADIUS $>$ 0 $\rightarrow$ external radius (defines x=0) RADIUS $<$ 0 $\rightarrow$ internal radius (defines x=0) For solid cylinders and spheres, always use RADIUS $>$ 0 because of division by the area, AREA1, which is zero for RADIUS $<$ 0.	0.	51–60

Notes: 1. SODDIT can internally adjust the time-step size for time differencing based on various criteria. The range of admissable values is given by DTMIN $\leq$ DTIME $\leq$ DTMAX. Among the criteria are that, for $\theta \neq 1$, the coefficients of the matrix equations must all be positive (the usual criterion for numerical stability), that the temperature of certain nodes should not change by more than DTEMPM in a time step, or that the next future problem time should match the print-time (TPRNT). To have a fixed-time step, set DTMAX = DTMIN.

2. THETA can be any number between 1. and 1.E-20. If it is less than 1.E-20, for example blank, a value of 1. is used.

3. The time units of the problem are primarily determined by the units of the Stéfan-Boltzmann constant, σ, the input thermal conductivity; and/or the convective heat-transfer coefficient h. All of these quantities must have consistent time units.

Variable	Description	Default Value	Record Columns
EXPN	Exponent used to generate the area and volume as function of position within the problem domain. Automatic area computation occurs for EXPN = 0.,1.,2. For other area variations, the area data must be entered in tabular form. EXPN = 0. or blank → planar domain EXPN = 1. → cylindrical domain EXPN = 2. → spherical domain	0.	61–70
FRACT1	Fraction of surface element remaining before it is combined with next element and surface node is dropped; applicable only for IBCTY1=1 problems with ablation.	1.0E-03	71–80

Block 5: Material Property Data

$$\left(\sum_{j=1}^{MAT} i_j \text{ records}\right)$$

record 1: RHO, DHF, TREFF, MATNAM
FORMAT (3F10.0, 10A4)
record 2, 3,--i_j: IFLG, TT, TCP, TK, TEMIT
FORMAT (I2, 4F10.0)

where

MAT = total number of materials in the problem domain; MAT $\leq$ 10.
i_j = total number of entries in j-th material table; $i_j \leq 30$.

The material property data needed for heat diffusion problems are density (ρ), specific heat (C_p), thermal conductivity (k), and emittance (ϵ). In SODDIT, density is assumed to be independent of temperature while specific heat, thermal conductivity, and emittance are treated as tabulated functions of temperature. For each temperature entry (TT) in a material's table, values of C_p (TCP), k(TK), and ϵ (TEMIT) must be included on the same record. Each material table can contain no more than 30 entries (records) and there can be no more than 10 separate material tables (i.e., MAT $\leq$ 10). These limits are determined by DIMENSION statements and can be readily changed. Linear interpolation is used within each property table. As indicated, multiple material problems are easily handled by simply repeating the above process for each material, placing each set of records one after the other. The materials are numbered by SODDIT in the order in which they are read, with the first material properties entered being material 1. The material number is used to identify the material in which energy sources are located when more than one material comprises the problem domain (see Block 11, NMAT).

The following table describes the input variables and their location on a record for a *typical* material. Repeat this sequence for multiple materials.

The j-th ($j \leq 10$) Material Property Table Input (for a gap, read Note 4 below)

Variable	Description	Record Number	Record Column
RHO	Density of the j-th material, in consistent units.	1	1–10
DHF	Heat of formation of the j-th material. Ignore for nonablating problems.	1	11–20
TREFF	Reference temperature for DHF [consistent units or KR(19) $\neq$ 0]. Leave blank for nonablating problems.	1	21–30
MATNAM	Alpha/numeric characters describing the j-th material. Blanks are acceptable.	1	31–70
For each temperature entry (maximum of 30), the following is required:			
IFLG	integer flag used to terminate the material input process. IFLG = 0 more information to follow in this table. IFLG = −1 last entry in j-th table, but there are more material property tables to follow. IFLG = +1 last entry of the last material property table.	$2–i_j$	1– 2
TT	Tabular temperature entry. If the problem contains thermal radiation, TT must be in consistent (absolute) units, or else KR(19) must be used.	$2–i_j$	3–12
TCP	Tabular specific heat entry in consistent units.	$2–i_j$	13–22
TK	Tabular thermal conductivity entry in consistent units.	$2–i_j$	23–32
TEMIT	Tabular emittance entry. This can be ignored if radiation to the surroundings is not important; for example, an interior material layer of a composite body. However, if this material is on either side of a radiation gap, it should be nonzero.	$2–i_j$	33–42

Notes: 1. Since the program uses linear interpolation on temperature, at least two entries must be supplied in every table.

2. Uniform (or non-temperature dependent) properties can be input for a material by simply entering the same value of the property for each temperature entry. In this case, only two records are needed.

3. SODDIT will extrapolate property values outside the range of input temperature. To avoid extrapolation problems, the TT entries should span the temperature range of the problem.

4. If any "gaps" (see Section 1, Interface Conditions, for a discussion of gaps) are present in the grid, then the material property table for the gap element must be reinterpreted. From the discussion in Section 1, the input gap properties should include Nu, k, and h_{cont}. Rather than altering the input data format for this situation, the existing material property table format will be utilized and the input will be reinterpreted. Although Record Number 1 is not utilized for a "gap," it must still be input; all blank fields are satisfactory. The flag IFLG is still used to signal the end of a table. The tabular temperature TT is still the independent variable. The variable TCP now becomes the dimensionless Nusselt number, Nu. The tabular thermal conductivity (TK) is the thermal conductivity of the *gap* medium. The tabular emittance TEMIT becomes the contact conductance h_{cont}. In summary,

   TT = tabular temperature
   TCP = Nu based on gap width and minimum area
   TK = thermal conductivity of gap medium
   TEMIT = contact conductance, h_{cont}

   Radiation properties of gap surfaces are obtained from adjoining material property tables.

Block 6-A: Nodal and Element Data – Automatic Node Generation: KR(11)=3

(NR+2 records)

record 1: TINIT, consistent units or use KR(19) $\neq$ 0
FORMAT (F10.0)
record 2: NMATR(I), NER(I), XR(I), DX1(I), DXF(I), for subregion I = 1, consistent units or use KR(10) $\neq$ 0
FORMAT (2I5, 4F10.0)
record 3: Same as record 2 but for subregion I = 2.
•
•
•
record NR+1: Same as previous record but for subregion I = NR
record NR+2: Blank (or zero) record, used to signal termination of this input section.

TINIT = initial temperature of the entire problem domain, and must be a constant for this option. If the initial temperature varies with position, BLOCK 6-B must be used instead of BLOCK 6-A.

NMATR = material property number of elements located in the subregion XR. MATR = 0 or blank is used as a terminator for this data block.

NER = number of elements in region XR. (NER = −1 designates a radiation/convection gap or the presence of a contact conductance).

XR = thickness of subregion of problem domain.

DX1 = thickness of first element of region XR.

DXF = thickness of the last element in region XR.

SUBROUTINE ANG (i.e., Automatic Node Generation) will construct the nodal and element information by dividing the domain into NR $\leq$ 8 subregions, of thickness XR(I) and NER(I) elements of material number NMATR(I) and locating nodes at the ends of each element in each subregion, and with element thicknesses being determined by the geometric progression equations given in Section 1, Automatic Grid Generation Using Geometric Progression. Present dimension limitations restrict the total number of elements NTE $\leq$ 100. This routine also counts the number of subregions to determine NR, initializes all nodal temperatures to TINIT, and automatically accumulates nodes and elements so that the number of total nodes (NTN) and number of total elements (NTE=NTN−1) are defined. The final feature of ANG is to automatically set the proper flags such that the front- and back-face nodal temperatures of each subregion are saved for a subsequent summary printing [KR(24)=0].

Records 2 through NR-1 contain the variables NMATR, NER, XR, DX1, and DXF. Under no circumstances will it be necessary to input all of the last four variables; at most, only three will be required and the fourth will be calculated internally. In the case of uniform thickness elements within a subregion, only two variables will be required. Figure 6 presents a summary of the four options that are available. These options correspond to Case A and Case B discussed in Section 1 and two suboptions, both of which are related to a uniform element thickness within each subregion.

CASE A (NER, XR, DXI)

The fourth variable DXF is left blank and will be calculated internally by the code. The code assumes that the user wants an expanding grid (DXF > DX1, K > 1.0); for a contracting grid, use CASE B below. See Section 1 for a discussion of the equations used to calculate DXF.

CASE A.1 (NER, XR)

This option produces uniform element thickness within this subregion; however, other subregions can still have a nonuniform thickness. Both the quantities DX1 and DXF are set equal to XR/FLOAT(NER).

CASE B (XR, DX1, DXF)

The fourth variable NER is left blank and is calculated from the three input quantites; see Section 1 for a discussion of the appropriate equations. The grid can be expanding or contracting depending on the direction of the inequality between DX1 and DXF. Since NER is calculated, it is possible for the total number of elements NTE to exceed the dimensioned value of 100; ANG checks NER and if it exceeds 100, then execution is terminated.

CASE B.1 (DX1, XR)

Uniform thickness elements can also be produced by leaving both NER and DXF zero or blank. For this option, NER = INT(XR/DX1 + 0.5) and DX1 is then replaced by XR/FLOAT(NER).

CASE	NMATR	NER	XR	DX1	DXF	COMMENTS
A	x	x	x	x	b	Geometric progression, DXF calculated internally
A.1	x	x	x	b	b	Equal elements, DX1 = DXF = XR/FLOAT (NER)
B*	x	b	x	x	x	Geometric progression, NER calculated internally
B.1*	x	b	x	x	b	Equal elements, DX1 = DXF, NER = INT (XR/DX1 + 0.5) DX1 = XR/FLOAT (NER)

x signifies non-zero input quantity

b signifies variable left blank or input as zero

* the value of NER calculated with this option may exceed the dimension limit of 100 elements; use this option with caution.

Figure 6. Schematic of Input to SUBROUTINE ANG

Block 6-B: Nodal and Element Data – Individual Nodal Input

[NTN records]

record j, j = 1,2,. . .,NTN: IFLG, NODE, NMAT, DXI, T, AREAI, VOLI, NGAP
FORMAT (I1, I4, I5, 4F10.0, I5)

For problems that have a variable cross-sectional area, or that have a nonuniform initial temperature distribution or a desired variable element size distribution, this block of input data *must* be used instead of Block 6-A. The following table describes the variables and their input location on a typical record.

Variable[1]	Description	Columns
IFLG	Integer flag used to control input of this section. IFLG = 0 more input data are to come. IFLG ≠ 0 final record of input.	1
NODE=J	Node number. Useful for organizing input, but is not used for computation in SODDIT. Thus, it may be left blank.	2–5
NMAT(J)	Material number of *element* J	6–10
DXI(J)	Thickness of *element* J in consistent units. If DXI(J) = 0., then DXI(J) = DXI(J−1), so if DXI is to be a uniform value throughout the domain, only a nonzero value on the first record of this Block is needed to generate the element distribution.	11–20
T(J,1)[2]	Initial temperature of *node* (not *element*) J, in consistent units, or use KR(19).	21–30 31–40
AREAI(J)[3]	J-th *element* area normal to the heat flow vector, in consistent units.	
VOLI(I)[3]	J-th *element* volume, in consistent units.	41–50
NGAP(J)	Integer flag to indicate that this *element* has an energy transfer gap in it. This allows for either a finite contact conductance between materials or a radiative/convective exchange between two materials. NGAP(J) = 0 no radiative/convective gap or contact conductance present. NGAP(J) = −1 a radiative/convective gap or contact conductance is present in this element. A special material property table must be used for this element (see Block 5).	51–55

Notes: 1. The maximum number of entries in this Block is 101; there can be 100 elements and 101 nodes. This limit can be changed by altering the appropriate DIMENSION statements.

2. Since the number of nodes will always be one more than the number of elements, the last record in this Block should contain only IFLG and T.

3. If the user wants to be consistent with the area and volume calculations in subroutine ANG, then the equations in Table 2 should be used. However, the user should feel free to experiment with other geometrical calculation procedures.

Block 7: Boundary Condition Constants

(2 records)

record 1: AWALL1, TRAD1, IBCTA1, IBCTY1, NINVN, NFT, NDTTQ, IFST, ISKP
FORMAT (2F10.0, 715)
record 2: AWALLN, TRADN, IBCTAN, IBCTYN
FORMAT (2F10.0, 715)

Front-Face (x = 0) Boundary Condition Constants: Record 1

Variable	Description	Default Value	Record Columns
AWALL1	Absorptivity of the surface material at node 1, i.e., α_w. If AWALL1 is input as $\leq$ 1.E-6 (i.e., as zero) then SODDIT assumes $\alpha_w = \epsilon_w$ for the front face and uses the temperature-dependent value from the appropriate material property table.	none	1–10
TRAD1	Temperature with which node 1 exchanges radiation, cf, Eq. (1-4). If KR(14) $\neq$ 0, then it is assumed that $T_{rad} = T_\infty$, where T_∞ is found in the front face boundary condition table of Block 8-D; consistent units or use KR(19) $\neq$ 0.	none	11–20
IBCTA1	Number of the front-face boundary condition table. This entry can be ignored at present.	1	21–25
IBCTY1	Index of the boundary condition type for the front face. There are four options: IBCTY1 = 1 specifies a general convection/radiation condition, cf, Eq. (1-3). IBCTY1 = 2 specifies the surface temperature history IBCTY1 = 3 specifies a general convection/radiation condition, cf, Eq. (1-5). IBCTY1 = 4 indicates the problem is an *inverse* problem, and the boundary flux is to be estimated by SODDIT.	1	26–30

Ignore (i.e., leave blank) the remaining part of this record for direct problems.

Variable	Description	Default Value	Record Columns
NINVN[1]	Number of nodes for which input temperature histories for inverse computations are specified (i.e., number of sensors) NINVN $\leq$ 7.	none	31–35
NFT[1]	Number of future times (in the temperature sensor tables of Block 8-I) used to estimate the boundary flux.	1	36–40
NDTTQ[1]	The number of times the sensitivity coefficients are computed between two consecutive times in the sensor history tables of Block 8-I, $\Delta t_x = \Delta t/NDTTQ$. NDTTQ $\geq$ 1. Suggest the user perform numerical experiments with NDTTQ = 1 and 2 and compare results.	1	41–45
IFST[1]	Index of the first time entry in the front-face boundary condition table of Block 8-I to be used in the inverse calculations. This allows sensor data to be skipped if the initial (IFST-1) entries are essentially unchanged.	1	46–50
ISKP[1]	Increment by which the time index in the sensor table is advanced. This allows selected data to be skipped without destroying the original data file. ISKP = 1 every data point is considered ISKP = 2 every second data point is considered • • •	1	51–55

Back-Face (x = L) Boundary Condition Constants (same as Record 1 of this block but for the back-face boundary condition): Record 2

Variable	Description	Default Value	Record Columns
AWALLN	Absorptivity of the surface material at node NTN, i.e., α_w. If AWALLN is input as $\leq$ 1.E-6 (i.e., as zero) then SODDIT assumes $\alpha_w = \epsilon_w$ for the back face and uses the temperature-dependent value from the appropriate material property table.	none	1–10
TRADN	Temperature with which node NTN exchanges radiation, cf, Eq. (1-4). If KR(14) $\neq$ 0, then it is assumed that $T_{rad} = T_\infty$, where T_∞ is found in the back-face boundary condition table of Block 9; consistent units or use KR(19) $\neq$ 0.	2	21–25
IBCTAN	Number of the back-face boundary condition table. This entry can be ignored at present.	2	21–25
IBCTYN	Index of the boundary condition type for the back face. There are only two options: IBCTYN = 2 specifies the surface temperature boundary condition IBCTYN = 3 specifies a general convection/radiation boundary condition; cf, Eq. (1-5).	1	26–30

Note: 1. This variable is needed *only* for inverse problems and may be ignored (blank entry made) for direct problems.

BLOCK 8-D: Front Face (x = 0) Boundary Condition Table – Direct Problems

(NFTE records)

record i, $1 \leq i \leq$ NFTE: IFLG, TTIME, THR, TQRAD, TCH
FORMAT (I2, F8.0, 4F10.0)

This table contains a maximum of 600 records, i.e., NFTE $\leq$ 600 (NFTE = number of front-face table entries) and the table is terminated by placing "−1" in columns 1 and 2 of the last record of the table. In this manner, time-dependent boundary conditions can be modeled in a tabular format. *All* problems must have *at least two* records since linear interpolation in time is used.

Variable	Description	Record Columns
IFLG	Flag that controls the number of entries in the front-face boundary condition table. IFLG = 0 indicates more records are to follow. IFLG = −1 indicates the last record of this table.	1–2
TTIME	Time for the i-th record's data, consistent units.	3–10
THR	This variable plays three roles, depending upon IBCTY1 (Record 7). IBCTY1 = 1 indicates that THR is the recovery enthalpy, i_r, used in convection at the i-th tabulated time, consistent units. IBCTY1 = 2 indicates that THR is the surface temperature at the i-th tabulated time. IBCTY1 = 3 indicates that THR is the environmental temperature, T_∞, used in convection [or radiation if KR(14) ≠ 0], at the i-th tabulated time. THR must be in consistent units; KR(19) ≠ 0 is *not valid* for this variable.	11–20
TQRAD	This variable plays two roles depending upon IBCTY1. In both cases, the entry is for the i-th time. IBCTY1 = 1 indicates a known incident radiation flux history, consistent units. IBCTY1 = 2 indicates a known surface recession rate history, consistent units. IBCTY1 = 3 indicates a known incident radiation flux history, consistent units.	21–30

Variable	Description	Record Columns
TCH	This variable is used only for IBCTY1 = 1 or 3. In all other cases, it can be ignored. It is the known convection heat transfer coefficient history, consistent units. IBCTY1 = 1 indicates C_h based on enthalpy difference driving potential. IBCTY1 = 3 indicates h based on temperature difference driving potential.	31–40
TPR	This variable is used only for IBCTY1 = 1 and can be ignored in all other cases. It is the gas pressure adjacent to the front-face boundary. The units are arbitrary provided they are consistent with the pressure units of Block 10, Gas Enthalpy Data.	41–50

Block 8-I: Front Face (x = 0) Boundary Condition Table – Inverse Problems

[1 + NFTE records]

record 1: NINV(I), I = 1, NINVN, NINVN $\leq$ 7
FORMAT (7I5)
record 2, 3,. . .NFTE + 1: IFLG, TTIME, Y(J,K), where 1 $\leq$ J $\leq$ NFTE, NFTE $\leq$ 600 and 1 $\leq$ K $\leq$ NINVN
FORMAT (I2, F8.0, 7F10.0)

The first record of this block contains the node numbers where temperature histories (i.e., sensor data) are known (in tabular form). The grid *must* be designed so that sensor locations coincide with the grid nodal locations; the user is cautioned to carefully select input information to SUBROUTINE ANG. A maximum of seven temperature sensors are permitted. There may be more than one sensor at a given node location (i.e., depth). If so, the node number must be repeatedly entered as many times as necessary.

The next NFTE (number of front face table entries) records ($\leq$600) contain the temperature history data. For the J-th record (i.e., the J-th time entry in the history data) the input is:

Variable	Description	Record Columns
IFLG	Index controlling input of sensor data. IFLG = 0 indicates more records follow IFLG = −1 indicates the last record	1–2
TTIME	Time of the J-th entry for sensor data, consistent units.	3–10
Y(J,1)	Temperature of sensor 1 at time J, consistent units or KR(19) $\neq$ 0.	11–20
•	•	•
•	•	•
•	•	•
•	•	•
•	•	•
Y(J,NINVN)	Temperature of sensor NINVN ($\leq$7) at time J.	71–80

Block 9: Back Face (x = L) Boundary Condition Table

(NBTE records)

record i, i = 1, 2, . . ., NBTE: IFLG, TTIME, THR, TQRAD, TCH
FORMAT (I2, F8.0, 3F10.0)

This section inputs the boundary condition history table for the back face, x = L. There are NBTE (number of back-face table entries) records with NBTE $\leq$ 600. The following table describes the i-th record of input. NBTE must be greater than or equal to 2 because linear interpolation is used.

Variable	Description	Record Columns
IFLG	Indicator controlling input of back-face boundary condition data. IFLG = 0 indicates more back-face data follows. IFLG = +1 indicates the last record of the table.	1–2
TTIME	Time for the i-th entry in the back-face boundary condition table, consistent units.	3–10
THR	This variable can be one of two quantities depending on IBCTYN. IBCTYN = 2 means THR is the tabular surface temperature. IBCTYN = 3 means THR is the tabular environment temperature for convection (i.e., T_∞). Must be in consistent units, since KR(19) $\neq$ 0 is *not valid* for this variable.	11–20
TQRAD	This is the tabulated radiation flux incident on the back face (cf, Eq. 1-4). TQRAD can be ignored except for IBCTYN = 3.	21–30
TCH	This is the tabulated convection heat transfer coefficient based on ($T_s - T_\infty$). TCH can be ignored except for IBCTYN = 3, consistent units.	31–40

Block 10: Gas Enthalpy Data (for gas adjacent to front face) (skip this section if IBCTY1 ≠ 1.)

$$\left(\sum_{i=1}^{NPT} NT_i \text{ records}\right)$$

When the convective heat-transfer rate at the front face is given by a recovery-enthalpy-difference driving potential times a transfer coefficient C_h (Type 1 boundary condition, IBCTY1 = 1), the enthalpy of the gas adjacent to the front face must be specified. SODDIT allows the enthalpy to be input as a tabular function of both pressure and temperature, and two-dimensional linear interpolation using SUBROUTINE LOOK2D is used. The tables are arranged in NPT (= no of pressure tables) subblocks for each given pressure. For a given pressure, the table contains enthalpy as a function of temperature, and the NT_i (= no of temperature entries in i-th pressure table) temperature entries must be in either ascending or descending order. A control flag, IFLG, is used to indicate the last temperature entry in a given pressure table and also the last record of this block. This frees the user from having to count the records in the various pressure tables. It is not necessary to have the same number of pressure records in each table. The present dimension statements limit the number of pressure tables to a maximum of 10 with no more than 25 temperature entries in a given pressure table. Since linear interpolation is used on both temperatures and (natural log of) pressure, a minimum of two pressure tables are required, each with a minimum of two temperature entries; a pressure of 0.0 is not acceptable. For a problem in which the heat capacity of the gas is a constant, then only two temperature entries are necessary. The pressure tables must be in either ascending or descending order. For the i-th temperature entry in the j-th pressure table, the input is as follows:

Variable	Description	Record Columns
TP	Tabular pressure, in the same units as TPR of BLOCK 8-D.	1–8
TBP	Tabular B′, dimensionless mass-loss-rate. For nonablating problems, set this to a number less than or equal to 1.0E-9.	17–24
TTS	Tabular surface temperature, in absolute units. For TTS in R, enter with a negative sign; for TTS in K, enter with a positive sign. Note that SODDIT assumes that the problem temperature units are R for this option and will convert TTS to R if it is input as K.	25–33
THW	Tabular wall enthalpy of gas adjacent to front face. If TTS is in R, then this will be in Btu/lbm; if TTS is in K, then this will be in cal/gm.	48–56
IFLG	A flag used to indicate the end of a pressure table. IFLG = 0 read another temperature entry for the same pressure. IFLG = 1 last temperature entry of this pressure table but another pressure table will follow. IFLG = 3 last temperature entry of last pressure table, end of record.	80

The above procedure is repeated for each additional pressure and temperature combination; see Table 21 for an example problem of this type.

If KR(31) ≠ 0, then this block must also contain the heat of ablation and ablation temperature; otherwise, ignore.

QSTAR	Energy per unit mass consumed in the ablation process, consistent units.	1–10
TABL	Temperature of the surface when ablation takes place.	11–20

Block 11: Energy Generation Rate Constants. [Skip this section if KR(30) = 0.]

[1 record]

record 1: QA, QB, QC, QD, QE, MATGEN
FORMAT (5F10.0, I5)

If no generation is present in the problem [KR(30) = 0], this record can be ignored. The input is then complete. Should generation be present [KR(30) ≠ 0], values of QA, QB, QC, QD, QE, and MATGEN must be entered. MATGEN is the material number as numbered in the material properties table, corresponding to the material in which energy generation occurs, allowing only a *single* material of a composite to have energy generation. The default value is 1. SODDIT automatically numbers the materials with 1 being the first subtable of materials given, continuing with 2, 3, etc., until all material of the [0,L] domain have been entered.

There are at least four types of generation rate functions allowed. These options are flagged by KR(30):

KR(30) = 0 means no generation in the problem.

KR(30) = 1 means that the generation in material number MATGEN is uniform within its domain. Set QA = magnitude of the thermal source, [consistent units, energy/(time·volume)].

$QB = QC = QD = QE = 0.$

KR(30) = 2 means the generation is temperature dependent and of Arrhenius form, i.e.,

$\dot{Q} = \dot{Q}(T) = QA \exp(-QB/T)$

$QC = QD = QE = 0.$

Note that QB must have consistent temperature units.

KR(30) = 3 means the generation is temperature dependent and of the form:

$\dot{Q} = QA + QB*T + QC*T^2$

where QA, QB, and QC have consistent units and QD = QE = 0.

KR(30) = 4 means the generation is position dependent of the form

$\dot{Q}(t) = I_o\,(1-R)\mu e^{-\mu x}$

where the constants $QA = I_o$, $QD = \mu$, QE = R, QB, and QC are available to define $I_o(t)$ by means of FORTRAN coding. The units of I_o are energy/(time-area), and μ is reciprocal length. Since the volumetric energy generation rate depends on position, the following average element value is used in SODDIT:

$$\bar{\dot{Q}}_{(j)} = -\frac{I_o(1-R)}{x_{j+1} - x_j}\left[\exp(-\mu x_{j+1}) - \exp(-\mu x_j)\right] .$$

With this record entered, the data input for this problem is complete. Note that more than one problem can be considered in one execution of SODDIT by simply stacking individual problem input data files one behind another. No special control command is needed to signal the number of problems treated. SODDIT terminates execution when an "EOF" or "EOB" (indicates "end of file" or "end of buffer") is read. EOF or EOB is automatically placed at the end of all files.

3. The Difference Equations

Formulation

In this section, details pertaining to the development of the finite difference equations solved by SODDIT are presented. It has been shown in Section 1 that both direct and inverse problems require solutions to governing equations of identical form, i.e., Eqs. 1 through 8 of Section 1. Consequently, only one systematic development of the difference equations is needed.

Consider a one-dimensional transient heat diffusion problem. Recall that the problem domain has been shifted such that the coordinates of the computational problem are $0 \leq x \leq L$, with $x = 0$ called the "front face" and $x = L$ called the "back face." The material occupying the domain [0,L] may be homogeneous or not, and it can even have gaps across which radiation and/or convection takes place. The basic philosophy of SODDIT is to divide the domain into NTE discrete, contiguous elements with a node located at each element end as shown in Figure 7. Nodes are locations of the unknowns or dependent variables. Note that element numbers are in parentheses and will appear as superscripts. Further, SODDIT presumes that each element contains only one material so that each element has a single value for material properties, thickness, cross-sectional area, volume, and generation rate. The temperature-dependent element properties are evaluated at the arithmetic average temperature of the element at the previous time step. Experience has shown that this linearization is adequate for solving a wide range of engineering problems; thermal properties are seldom known to sufficient accuracy to justify the added computational effort required by iteration at each time step.

The next step is to superimpose a series of NTN = NTE + 1 control volumes atop the domain, as illustrated in Figure 8 for a typical volume. Notice that the control volume boundaries correspond to the element equivalent radius given in Table 2. Within each element, a linear temperature profile is assumed of the form

$$T^{(j-1)}(x,t) = [T_{j-1}(t)(x_j - x) + T_j(t)(x - x_{j-1})]/(x_j - x_{j-1}) \qquad (3\text{-}1)$$

$$x_{j-1} \leq x \leq x_j$$

where the nodal values of T are assumed, in general, to depend upon time.

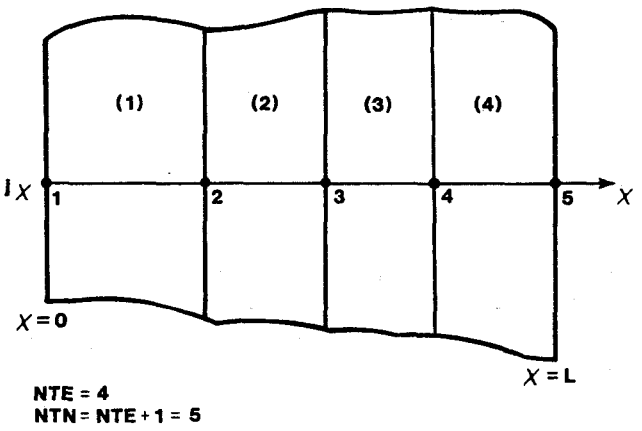


Figure 7. A Material Divided Into Four Elements With Five Nodes. (NTE = number of elements, NTN = number of total nodes).

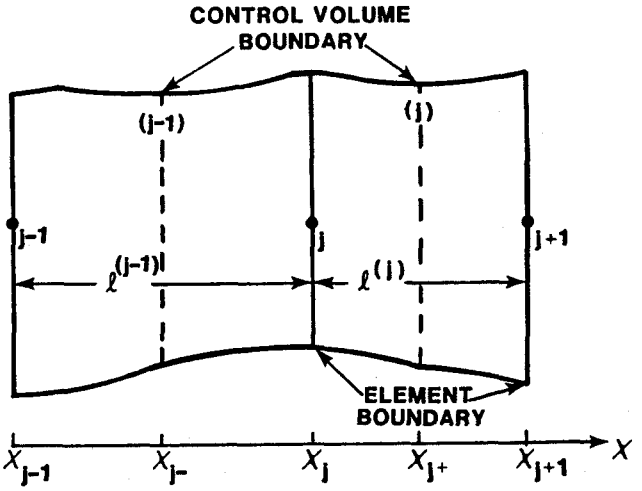


Figure 8. A Typical Control Volume for an Internal Location. The volume surrounds node j and includes contributions from elements (j−1) and (j).

The linear temperature profile was chosen for several reasons, including simplicity and interelement continuity. For constant thermal properties, planar geometry, and no sources, the steady-state temperature profile will be linear. Consequently, steady-state

temperature profiles will be calculated very accurately. If approximate analytical solutions are available for the element temperature profile for situations other than steady state, the procedure described below can be readily adapted to handle such cases.

The goal is to enforce conservation of energy over the control volume in a weighted average sense. That is, if L(T) = 0 represents the governing equation (1-1) for this problem and knowing that Eq. (3-1) is an approximation to the true temperature (i.e., $T^* \simeq T$), then it is required that

$$\int_{x_{j-}}^{x_{j+}} \omega(x) L(T^*)\, A\, dx = 0 \ ,$$

$$L(T) = \rho C_p(T)\frac{\partial T}{\partial t} - \frac{1}{A}\frac{\partial}{\partial x}\left[k(T)A\frac{\partial T}{\partial x}\right] - \dot{Q}(T) = 0 \qquad (3\text{-}2)$$

even though T^* only rarely identically satisfies L(T) = 0. The functions $\omega(x)$ are called weighting functions. Their specific form is left to the user; SODDIT allows three forms for $\omega(x)$:

Lumped Capacitance Method (LC):

$$\omega(x) = \begin{cases} \delta(x-x_j)^{\dagger} \ , & x_{j-} \le x \le x_{j+} \\ 0 \ , & \text{elsewhere} \end{cases}$$

Finite Control Volume Method (FCV):

$$\omega(x) = \begin{cases} 1, & x_{j-} \le x \le x_{j+} \\ 0 \ , & \text{elsewhere} \end{cases} \qquad (3\text{-}3)$$

Finite Element Method (FE):

$$\omega(x) = \begin{cases} (x-x_{j-1})/(x_j-x_{j-1}) \text{ if } x_{j-} \le x \le x_j \\ (x_{j+1}-x)/(x_{j+1}-x_j) \text{ if } x_j \le x \le x_{j+} \\ 0, \text{ elsewhere} \end{cases} .$$

The LC approach is also called the collocation method while the FCV method is also known as the subdomain method, and the FE method uses the Galerkin method for extracting nodal equations. In the next subsection, the FCV method will be used to develop the difference equations used in SODDIT.

*Where $\delta(x - x_j)$ is the Dirac delta function $= \begin{cases} 0 \text{ if } x \neq x_j \\ 1 \text{ if } x = x_j \end{cases}$

Nodal Equations for Interior Nodes

Consider the typical internal control volume shown in Figure 8. The general governing principle for energy diffusion within this volume is [cf, Eq. (1-1)]:

$$\rho C_p(T)\frac{\partial T}{\partial t} = \frac{1}{A(x)}\frac{\partial}{\partial x}\left[k(T)A(x)\frac{\partial T}{\partial x}\right] + \dot{Q}(T) \ , \qquad (3\text{-}4)$$

wherein $\dot{Q}$ is the general expression for energy sources that may or may not depend on temperature T. Applying the FCV method to Eq. (3-4) gives

$$\int_{x_{j-}}^{x_j}\left[\rho C_p\frac{\partial T}{\partial t}A\right]^{(j-1)}dx$$

$$+\int_{x_j}^{x_{j+}}\left[\rho C_p\frac{\partial T}{\partial t}A\right]^{(j)}dx$$

$$=\int_{x_{j-}}^{x_j}\left[\frac{1}{A}\frac{\partial}{\partial x}\left[kA\frac{\partial T}{\partial x}\right]A\right]^{(j-1)}dx$$

$$+\int_{x_j}^{x_{j+}}\left[\frac{1}{A}\frac{\partial}{\partial x}\left[kA\frac{\partial T}{\partial x}\right]A\right]^{(j)}dx$$

$$+\int_{x_{j-}}^{x_j}[\dot{Q}A]^{(j-1)}dx$$

$$+\int_{x_j}^{x_{j+}}[\dot{Q}A]^{(j)}dx \qquad (3\text{-}5)$$

where

$$\frac{\partial T^{(j-1)}}{\partial t} = [\dot{T}_{j-1}(x_j-x) + \dot{T}_j(x-x_{j-1})]/(x_j-x_{j-1}) \ , \qquad (3\text{-}6)_1$$

$$\frac{\partial T^{(j)}}{\partial t} = [\dot{T}_j(x_{j+1}-x) + \dot{T}_{j+1}(x-x_j)]/(x_{j+1}-x_j) \ , \qquad (3\text{-}6)_2$$

$$\frac{\partial T^{(j-1)}}{\partial x} = (T_j - T_{j-1})/(x_j - x_{j-1}) \ , \qquad (3\text{-}6)_3$$

and

$$\frac{\partial T^{(j)}}{\partial x} = (T_{j+1} - T_j)/(x_{j+1} - x_j) \ . \qquad (3\text{-}6)_4$$

Note that $\dot{T}_{j-1} \equiv dT_{j-1}/dt$. The individual integrals in Eq. (3-5) can be thought of as element capacitance, conduction, and source contributions to each control volume. Note that two elements contribute to each control volume; or, each element contributes to two control volumes. Substituting Eq. (3-6) into Eq. (3-5), and with element properties assumed constant, gives

$$(\rho C_p \mathcal{V})^{(j-1)} \left(\frac{1}{8}\dot{T}_{j-1} + \frac{3}{8}\dot{T}_j\right) + (\rho C_p \mathcal{V})^{(j)} \left(\frac{3}{8}\dot{T}_j + \frac{1}{8}\dot{T}_{j+1}\right) = -\left(\frac{kA}{\ell}\right)^{(j-1)} (T_j - T_{j-1}) + \left(\frac{kA}{\ell}\right)^{(j-1)} (T_{j+1} - T_j) + \frac{(\dot{Q}\mathcal{V})^{(j-1)}}{2} + \frac{(\dot{Q}\mathcal{V})^{(j)}}{2} \quad (3\text{-}7)$$

wherein,

$\ell^{(j-1)} = x_j - x_{j-1}$ = length of element (j−1),

$\ell^{(j)} = x_{j+1} - x_j$ = length of element (j),

$\mathcal{V}^{(j)}$ = volume of element (j),

$(\ )^{(j-1)} \rightarrow$ evaluated for element (j−1),

$(\ )^{(j)} \rightarrow$ evaluated for element (j).

To solve this ordinary differential equation for T_j, finite differences in time are used. The time derivative is discretized by

$$\dot{T}_j \simeq \frac{T_j^{n+1} - T_j^n}{\Delta t} = \theta \dot{T}_j^{n+1} + (1-\theta)\dot{T}_j^n \quad (3\text{-}8)$$

where T_j^n is the value of T_j at time $n\Delta t$, T_j^{n+1} is the (unknown) future value of T_j at time $(n+1)\Delta t$, and θ is a weighting factor such that $0 \leq \theta \leq 1$. When θ is zero, the time differencing is called an *explicit* scheme; for $\theta = 0.5$, the partially implicit *Crank-Nicolson* scheme is used; and for $\theta = 1$, the differencing is *fully implicit*. The value of θ is an input variable; cf, Section 2, Block 4. The explicit scheme ($\theta = 0$) is not recommended; in fact, $\theta \geq 0.5$ is recommended. To implement Eq. (3-8), evaluate Eq. (3-7) at times (n+1) and at n, multiply by θ or $(1-\theta)$ as appropriate, and add the results. Whenever $\theta\dot{T}_j^{n+1} + (1-\theta)\dot{T}_j^n$—terms appear, replace them with the left side of Eq. (3-8). The results are

$$\left[\frac{C^{(j-1)}}{8} - K^{(j-1)}\theta\right] T_{j-1}^{n+1} + \left[\frac{3}{8}C^{(j-1)} + K^{(j-1)}\theta\right] T_j^{n+1} + \left[\frac{3}{8}C^{(j)} + K^{(j)}\theta\right] T_j^{n+1} + \left[\frac{C^{(j)}}{8} - K^{(j)}\theta\right] T_{j+1}^{n+1} = \left[\frac{C^{(j-1)}}{8} + K^{(j-1)}(1-\theta)\right] T_{j-1}^n + \left[\frac{3}{8}C^{(j-1)} - K^{(j-1)}(1-\theta)\right] T_j^n + \left[\frac{3}{8}C^{(j)} - K^{(j)}(1-\theta)\right] T_j^n + \left[\frac{C^{(j)}}{8} + K^{(j)}(1-\theta)\right] T_{j+1}^n + \left(\frac{\mathcal{V}^{(j-1)}}{2}\right) [\theta\dot{Q}^{n+1} + (1-\theta)\dot{Q}^n]^{(j-1)} + \left(\frac{\mathcal{V}}{2}\right)^{(j)} [\theta\,\dot{Q}^{n+1} + (1-\theta)\dot{Q}^n]^{(j)} \quad (3\text{-}9)$$

where $C^{(j)} = (\rho C_p \mathcal{V})^{(j)}/\Delta t$ and $K^{(j)} = (kA/\ell)^{(j)}$.

The terms involving $\dot{Q}$ are either known functions of space and/or time or else are known functions of temperature. In the first instance, the source terms are known and simply constitute additions to the right-hand side only; there is no explicit temperature effect. However, if $\dot{Q}$ depends on temperature, then explicit reference to T^{n+1} and T^n arise and the last two terms of Eq. (3-9) need to be reevaluated.

Consider that $\dot{Q} = \dot{Q}[T(x,t)]$. Then, in a given element (i.e., fixing x),

$$\dot{Q}^{n+1} = \dot{Q}^n + \left.\frac{d\dot{Q}}{dt}\right|_n \Delta t + \ldots, \quad \dot{Q}^n \equiv \dot{Q}(T^n)$$

but

$$\left.\frac{d\dot{Q}}{dt}\right|_n = \left.\frac{d\dot{Q}\,dT}{dT\,dt}\right|_n \simeq \left.\frac{d\dot{Q}}{dT}\right|_n \frac{(T^{n+1} - T^n)}{\Delta t},$$

giving

$$\dot{Q}^{n+1} \simeq \dot{Q}^n + \left.\frac{d\dot{Q}}{dT}\right|_n (T^{n+1} - T^n) .$$

Substituting this expression into the control volume integrals gives

$$\int_{x_{j-}}^{x_j} [\theta\dot{Q}^{n+1} + (1-\theta)\dot{Q}^n]^{(j-1)} A^{(j-1)} dx$$

$$+ \int_{x_j}^{x_{j+}} [\theta\dot{Q}^{n+1} + (1-\theta)\dot{Q}^n]^{(j)} A^{(j)} dx$$

$$= \int_{x_{j-}}^{x_j} \left[\dot{Q}^n A + \theta A \frac{d\dot{Q}}{dT}\bigg|_n (T^{n+1} - T^n)\right]^{(j-1)} dx$$

$$+ \int_{x_j}^{x_{j+}} \left[\dot{Q}^n A + \theta A \frac{d\dot{Q}}{dT}\bigg|_n (T^{n+1} - T^n)\right]^{(j)} dx$$

Now, using Eq. (3-1) for $T^{(j-1)}$ and $T^{(j)}$ and integrating gives, for the last two terms of Eq. (3-9),

$$\left[\frac{\mathcal{V}}{2}\dot{Q}(\overline{T}^n)\right]^{(j-1)} + \left(\theta\mathcal{V}_n \frac{d\dot{Q}}{dT}\bigg|\right)^{(j-1)}$$

$$\left[\left(\frac{1}{8}T_{j-1}^{n+1} + \frac{3}{8}T_j^{n+1}\right) - \left(\frac{1}{8}T_{j-1}^n + \frac{3}{8}T_j^n\right)\right]$$

$$+\left[\frac{\mathcal{V}}{2}\dot{Q}(T^n)\right]^{(j)} + \left(\theta\mathcal{V}\frac{d\dot{Q}}{dT}\bigg|_n\right)^{(j)}$$

$$\times\left[\left(\frac{3}{8}T_j^{n+1} + \frac{1}{8}T_{j+1}^{n+1}\right) - \left(\frac{3}{8}T_j^n + \frac{1}{8}T_{j+1}^n\right)\right]$$

where

$$(\overline{T}^n)^{(j-1)} = (T_{j-1}^n + T_j^n)/2$$

and

$$\frac{d\dot{Q}}{dT}\bigg|_n^{(j-1)} = \frac{d\dot{Q}}{dT}$$

evaluated at $\overline{T}^{(j-1)}$ and at time $n\Delta t$.

Therefore, the final control volume equation becomes:

$$\left[\frac{C}{8} - K\theta - \frac{\theta F}{8}\right]^{(j-1)} T_{j-1}^{n+1} + \left[\frac{3}{8}C + K\theta - \frac{3}{8}\theta F\right]^{(j-1)} T_j^{n+1}$$

$$+\left[\frac{3}{8}C + K\theta - \frac{3\theta}{8}F\right]^{(j)} T_j^{n+1} + \left[\frac{C}{8} - K\theta - \frac{\theta F}{8}\right]^{(j)} T_{j+1}^{n+1}$$

$$=\left[\frac{C}{8} + K(1-\theta) - \frac{\theta F}{8}\right]^{(j-1)} T_{j-1}^n$$

$$+\left[\frac{3}{8}C - K(1-\theta) - \frac{3}{8}\theta F\right]^{(j-1)} T_j^n +$$

$$\left[\frac{3}{8}C - K(1-\theta) - \frac{3}{8}\theta F\right]^{(j)} T_j^n$$

$$+\left[\frac{C}{8} + K(1-\theta) - \frac{1}{8}\theta F\right]^{(j)} T_{j+1}^n + \left[\frac{\mathcal{V}\dot{Q}(\overline{T}^n)}{2}\right]^{(j-1)}$$

$$+\left[\frac{\mathcal{V}\dot{Q}(\overline{T}^n)}{2}\right]^{(j)} \qquad \textbf{(3-10)}$$

where

$$F^{(j)} = \left(\mathcal{V}\frac{d\dot{Q}}{dT}\bigg|_n\right)^{(j)}$$

for the appropriate element. F is nonzero *only* if $\dot{Q}$ depends on temperature.

$$q_0''(t) \equiv -k\frac{\partial T}{\partial x}\bigg|_{x_0} = C_h[i_r - i_w(T_w)] + \alpha_{w_0} q''_{rad_0}(t)$$

$$-\epsilon_{w_0}(T_w)\sigma[T^4(x_0,t) - T_{rad_0}^4(t)] \; . \qquad \textbf{(1-3)}$$

Nodal Equations for Derivative Boundary Conditions

When a known flux is supplied to a boundary, the difference equations must be appropriately adjusted. For element (1) in Figure 7, a flux equal to $q_0''(t)$ is supplied to the boundary node number 1, the front face, where from Eq. (1-5), $q_0(t) = q_0''(t)A_0$ and, for a Type 3 boundary condition,

$$q_0(t) = h_0(t)A_0[T_{\infty_0}(t) - T(0,t)] + \alpha_{w_0} A_0 q''_{rad_0}(t)$$

$$-\epsilon_{w_0}(T)A_0\sigma\,[T(0,t)^4 - T^4_{rad_0}(t)] \qquad (3\text{-}11)$$

wherein A_0 is the area of the front surface at $x = 0$, *not* the element area $A^{(1)}$. A_0 is linearly extrapolated from $A^{(1)}$ and $A^{(2)}$, the element areas, or is evaluated from the appropriate analytical expressions. The procedure is to again integrate over the control volume centered on node 1, but in this case only half the volume is meaningful. Thus, the resultant nodal equation becomes

$$\left[3\frac{C}{8} + K\theta - 3\frac{\theta F}{8}\right]^{(1)} T_1^{n+1} + \left[\frac{C}{8} - K\theta - \frac{\theta F}{8}\right]^{(1)} T_2^{n+1}$$

$$= \left[\frac{3}{8}C - K(1-\theta) - \frac{3}{8}\theta F\right]^{(1)} T_1^n$$

$$+\left[\frac{C}{8} + K(1-\theta) - \frac{1}{8}\theta F\right]^{(1)} T_2^n + \left[\frac{\mathcal{V}\dot{Q}(\bar{T}^n)}{2}\right]^{(1)}$$

$$+ \theta q_0^{n+1} + (1-\theta)q_0^n\ . \qquad (3\text{-}12)$$

In this equation, q_0^n is Eq. (3-11) evaluated at time $n\Delta t$ (i.e., the present time) where all T's are known. On the other hand, q_0^{n+1} is the boundary-energy input rate evaluated at time $(n+1)\Delta t$ (i.e., the future time) where T_1 is unknown. Since q_0 depends on T in general, the value of q^{n+1} for Type 3 boundary conditions is based on linear-extrapolated (in time) surface temperatures, T_{sg} (the "guessed" value of the surface temperature). The extrapolation uses the values of surface temperature at time $n\Delta t$ and $(n-1)\Delta t$ to predict T_{sg} at $(n+1)\Delta t$.

The radiation term within $q_0(t)$ is linearized so that

$$q_0(t) = h_0A_0\,(T_{\infty_0} - T_1) + \alpha_{w_0} A_0\, q''_{rad_0} - h_{rad_0} A_0 \times$$

$$(T_1 - T_{rad_0}) \qquad (3\text{-}13)_1$$

where the subscript "0" denotes front face and h_{rad_0} is defined as

$$h_{rad_0}(T_1) = \epsilon_{w_0}(T_1)\sigma(T_1^2 + T^2_{rad_0})\,(T_1 + T_{rad_0})\ . \qquad (3\text{-}13)_2$$

Note that h_{rad_0} is always a known quantity since either T_1^n or T_{sg} is used to evaluate it. Thus $h^{n+1}_{rad_0}$ is only an approximation using $T_1^{n+1} = T_{sg}$. The approximation improves with decreasing time-step sizes. The quantity representing the last two terms on the right hand side of Eq. (3-12) becomes

$$\theta q_0^{n+1} + (1-\theta)q_0^n = -\theta A_0(h_0^{n+1} + h^{n+1}_{rad_0})T_1^{n+1}$$

$$+ q_{new_0} + q_{old_0}\ , \qquad (3\text{-}14)$$

where

$$q_{new_0} = \theta A_0[h_0^{n+1}\,T_{\infty_0}^{n+1} + h^{n+1}_{rad_0}\,T^{n+1}_{rad_0} + \alpha_{w_0}\,q''^{n+1}_{rad_0}]\ , \qquad (3\text{-}15)_1$$

and

$$q_{old_0} = (1-\theta)A_0\,[h_0^n(T_{\infty_0}^n - T_1^n) + h^n_{rad_0}\,(T^n_{rad_0} - T_1^n)$$

$$+ \alpha_{w_0}\,q''^n_{rad_0}]\ . \qquad (3\text{-}15)_2$$

The discretization equation of a derivative condition at the back face, i.e., at $x = L$ and element (j) = (NE), is identical to Eq. (3-14) and (3-15), but with the subscript "0" replaced with "L." The nodal values of T_1^{n+1} and T_1^n also become T^{n+1}_{NTN} and T^n_{NTN}, respectively.

In any case, the appearance of a derivative boundary condition causes the coefficient for either T_1^{n+1} or T^{n+1}_{NTN}, as appropriate, to be changed by adding the term "$\theta A_i(h_i + h_{rad_i})^{n+1}$" ($i$ = either 0 or L, or both) and the sum "$q_{new_i} + q_{old_i}$" to the right-hand-side terms of the control volume equations.

In the case where the front face is experiencing a Type 1 boundary condition, then $q_0(t) = q_0''(t)A_0$ where, from Section 1,

$$q_0''(t) \equiv -k\frac{\partial T}{\partial x}\bigg|_{x_0} = C_h[i_r - i_w(T_w)] + \alpha_{w_0}\,q''_{rad_0}(t)$$

$$-\epsilon_{w_0}(T_w)\sigma[T^4(x_0,t) - T^4_{rad_0}(t)]\ . \qquad (1\text{-}3)$$

Now the convective heat transfer is expressed in terms of the enthalpy difference of the fluid (usually a gas) between the ambient (recovery) state and the

wall. Since the enthalpy at the wall is a function of the wall temperature, T_w, and T_w is not known beforehand, iteration on T_w is required to find the value of T_w that satisfies Eq. (1-3).

The iteration algorithm is based on finding the value of T_w that forces, e, to be negligibly small, where, e, is defined to be the error in the surface energy balance:

$$e = \theta\, q''^{n+1}_{net} + (1-\theta) q''^{n}_{net} - q''_{cond} \qquad (3\text{-}16)_1$$

and where

$$q''_{net} = q''_{dif} + q''_{rad_{in}} - q''_{rad_{out}} \qquad (3\text{-}16)_2$$

$$q''_{dif} = C_h[i_r - i_w(T_w)] \qquad (3\text{-}16)_3$$

$$q''_{rad_{in}} = \alpha_{w_0}\, q''_{rad_0} \qquad (3\text{-}16)_4$$

$$q''_{rad_{out}} = \epsilon_{w_0}(T_w)\, \sigma[T_w^4 - T^4_{rad_0}(t)] \qquad (3\text{-}16)_5$$

and as discussed in Section 4, Front-Face Energy Balance and Temperature Profile Information,

$$q''_{cond} = (T_w/\omega'_1 - \gamma'_1)/A_1 \ . \qquad (4\text{-}13)$$

To find the root of $e(T_w) = 0$, a variant of Newton's method is used where

$$T_w^{p+1} = T_w^p - \frac{e(T_w^p)}{(de/dT_w)^p} \qquad (3\text{-}17)$$

where the superscript p denotes the current iteration step and T_w^{p+1} is the improved estimate of T_w. For each new time step, the initial estimate for T_w is extrapolated from the converged result for T_w from the prior time step. Extrapolation is done through a Taylor series in time. An analytical expression is used in SUBROUTINE SRFBNA to evaluate de/dT_w.

Convergence of Newton's method to an acceptible error tolerance, ϵ, is defined if the relative error, RE, after the current iteration satisfies

$$RE \le \epsilon \qquad (3\text{-}18)$$

where

$$RE = \max(RE_1, RE_2) \qquad (3\text{-}19)_1$$

and

$$RE_1 = |(e/q''_{cond})^p| \qquad (3\text{-}19)_2$$

$$RE_2 = |(T_w^{p+1} - T_w^p)/T_w^p| \ . \qquad (3\text{-}19)_3$$

Using both the error, e, and the wall temperature change between iterations insures that energy is balanced at the surface *and* the temperature computed at the boundary is the temperature of the boundary. This guards against cases wherein a small change in surface temperature causes a large change in the surface energy balance, and vice versa. Convergence usually occurs in fewer than three iterations for $\epsilon = 10^{-5}$.

General Form of the Discretization Equations

From the results of the control-volume-based discretization equations, Eqs. (3-10) and (3-14), a symmetric tridiagonal system of equations for T_j^{n+1} can be written of the general form

$$-A_j T_{j-1}^{n+1} + B_j T_j^{n+1} - A_{j+1} T_{j+1}^{n+1} = D_j \ , \qquad (3\text{-}20)$$

wherein, for $j = 2, 3, \ldots, NTN-1$,

$$-A_j = [\lambda(C - \theta F) - K\theta]^{(j-1)} \qquad (3\text{-}21)_1$$

$$B_j = \sum_{i=j-1}^{j} [(1/2 - \lambda)(C - \theta F) + K\theta]^{(j)} \qquad (3\text{-}21)_2$$

$$D_j = \sum_{i=j-1}^{j} d_i \ , \text{ where} \qquad (3\text{-}21)_3$$

$$d_{j-1} = [\lambda(C - \theta F) + (1 - \theta)K]^{(j-1)}\, T_{j-1}^n \qquad (3\text{-}21)_4$$
$$+[(1/2 - \lambda)(C - \theta F) - (1 - \theta)K]^{(j-1)}$$
$$\times T_j^n + \frac{[\nu\dot{Q}(\bar{T}^n)]^{(j-1)}}{2}$$

$$d_j = [(1/2 - \lambda)(C - \theta F) - (1 - \theta)K]^{(j)}\, T_{j+1}^n$$
$$+[\lambda(C - \theta F) + (1 - \theta)K]^{(j)}\, T_{j+1}^n$$
$$+ \frac{[\nu\dot{Q}(\bar{T}^n)]^{(j)}}{2} \qquad (3\text{-}21)_5$$

and for the boundaries,

$$A_1 = 0$$

$$B_1 = [(1/2 - \lambda)\,(C - \theta F) + K\theta]^{(1)} + \theta A_0\,(h_0 + h_{rad_0})^{n+1} \qquad (3\text{-}21)_6$$

$$D_1 = [(1/2 - \lambda)\,(C - \theta F) - (1 - \theta)K]^{(1)}\,T_1^n + [\lambda(C - \theta F) + (1 - \theta)K]^{(1)}\,T_2^n + [\mathcal{V}\dot{Q}(\overline{T}^n)]^{(1)}/2 + q_{new_0} + q_{old_0}$$

and

$$A_{NTN+1} = 0$$

$$B_{NTN} = [(1/2 - \lambda)(C - \theta F) + K\theta]^{(NTE)} + \theta A_L(h_L + h_{rad_L})^{n+1} \qquad (3\text{-}21)_7$$

$$D_{NTN} = [\lambda(C - \theta F) + (1 - \theta)K]^{(NTE)}\,T^n_{NTN-1} + [(1/2 - \lambda)(C - \theta F) - (1 - \theta)K]^{(NTE)}\,T^n_{NTN} + [\mathcal{V}\dot{Q}(\overline{T}^n)]^{(NE)}/2 + q_{new_L} + q_{old_L}\,.$$

The parameter λ allows the user to select the method of discretization, i.e., the weighting function. The following table summarizes the three possible values of λ available in SODDIT.

Table 4. Values of λ Available in SODDIT

Discretization Method	λ
Lumped Capacitance (LC)	0
Finite Control Volume (FCV)	1/8
Finite Element (FE)	1/6

It is noted that there is symmetry in the A-coefficients. Consider the following system of nodal equations for NTE = 3 and NTN = 4:

$$B_1T_1 - A_2T_2 + 0T_3 + 0T_4 = D_1$$

$$-A_2T_1 + B_2T_2 - A_3T_3 + 0T_4 = D_2$$

$$0T_1 - A_3T_2 + B_3T_3 - A_4T_4 = D_3$$

$$0T_1 + 0T_2 - A_4T_3 + B_4T_4 = D_4$$

or

$$\begin{bmatrix} B_1 & -A_2 & 0 & 0 \\ -A_2 & B_2 & -A_3 & 0 \\ 0 & -A_3 & B_3 & -A_4 \\ 0 & 0 & -A_4 & B_4 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix}^{n+1} = \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix}^{n} \qquad (3\text{-}22)$$

The system is clearly tridiagonal, and in addition, the value of A_2 multiplying T_2^{n+1} in the equation for T_1^{n+1} is identical to A_2 used to multiply T_1^{n+1} in the equation for T_2^{n+1}. This property is present due to the control volume being composed of half the volumes of two adjacent *elements*. Thus, as the control volume "moves" from node to node, it gathers information from parts of two elements: the one on the right-hand side of the node, and one on the left-hand side. Since each *element* has its own, uniform or average values of ρ, C_p, k, $\mathcal{V}$, $\dot{Q}$, and F, and since each element volume is involved with *two* nodal equations (or control volumes), it follows that the coefficient matrix is symmetric. This simplifies the solution method needed to solve the system of linear algebraic equations as well as the number of computations needed to complete the coefficient matrix of Eq. (3-22). The linear system of equations is solved using a variant of the Thomas algorithm for symmetric tridiagonal equations. If the user should attempt to expand the range of problems that SODDIT can handle, care should be taken to preserve the existing symmetry or rewrite the tridiagonal-equation-solver routines.

The Global Matrices

The general form of the discretization equations given in the previous section suggests an efficient method for assembling the global tridiagonal matrices of the complete problem, an example of which is illustrated in Eq. (3-22). It is recognized that the global system of linear equations will then be of the form

$$\underline{M}\,T = D \qquad (3\text{-}23)$$

where $\underline{M}$ is the square global "stiffness" matrix, D is the global "load" vector, and T is the vector of unknown nodal temperatures. The task, then, is to compute the elements of $\underline{M}$ and D in an efficient manner. It is noted that the discretization equations (3-10 and 3-14) result from an energy balance for a control volume. Each control volume spans two contiguous elements. With this background, the assembly algorithm can be developed.

Assembly

The expression developed earlier,

$$-A_j\ T_{j-1}^{n+1} + B_j\ T_j^{n+1} - A_{j+1}\ T_{j+1}^{n+1} = D_j \qquad \textbf{(3-20)}$$

represents a partial assembly of the matrix that, in turn, represents the contribution to the global problem of node j. By close examination, it is seen that Eq. (3-20) can be decomposed into contributions each from *elements* (j−1) and (j), which enclose node j (cf, Figure 8). To see this, carefully examine Eq. (3-10). Assuming that node numbers range from 1 to N (actually ISURF to NTN in SODDIT) and that there are NTE = N−1 elements, an expansion of this equation shows that

node 1:

$$B_1^{(1)}\ T_1^{n+1} - A_2^{(1)}\ T_2^{n+1} = D_1^{(1)}$$

node 2:

$$-A_2^{(1)}\ T_1^{n+1} + (B_2^{(1)} + B_2^{(2)})\ T_2^{n+1} - A_3^{(2)}\ T_3^{n+1} = D_2^{(1)} + D_2^{(2)}$$

•
•
•

node j−1:

$$-A_{j-1}^{(j-2)}\ T_{j-2}^{n+1} + (B_{j-1}^{(j-2)} + B_{j-1}^{(j-1)})\ T_{j-1}^{n+1}$$

$$- A_j^{(j-1)}\ T_j^{n+1} = D_{j-1}^{(j-2)} + D_{j-1}^{(j-1)}$$

node j:

$$-A_j^{(j-1)}\ T_{j-1}^{n+1} + (B_j^{(j-1)} + B_j^{(j)})\ T_j^{n+1} - A_{j+1}^{(j)}\ T_{j+1}^{n+1}$$

$$= D_j^{(j-1)} + D_j^{(j)}$$

node j+1:

$$-A_{j+1}^{(j)}\ T_j^{n+1} + (B_{j+1}^{(j)} + B_{j+1}^{(j+1)})\ T_{j+1}^{n+1} - A_{j+2}^{(j+1)} T_{j+2}^{n+1}$$

$$= D_{j+1}^{(j)} + D_{j+2}^{(j+1)}$$

•
•
•

node N:

$$-A_{N-1}^{(NTE)}\ T_{N+1}^{n+1} + B_N^{(NTE)}\ T_N^{n+1} = D_N^{(NTE)}\ .$$

Thus, collecting all quantities with a given *element* number, say element (j), which then include both T_j^{n+1} and T_{j+1}^{n+1}, it is possible to define element *matrices* of the form

$$\begin{bmatrix} B_j & -A_{j+1} \\ -A_{j+1} & B_{j+1} \end{bmatrix}^{(j)} \begin{Bmatrix} T_j^{n+1} \\ T_{j+1}^{n+1} \end{Bmatrix} = \begin{Bmatrix} D_j \\ D_{j+1} \end{Bmatrix}^{(j)} \qquad \textbf{(3-24)}_1$$

as, in general,

$$M^{(e)}\ T^{(e)} = D^{(e)}\ . \qquad \textbf{(3-24)}_2$$

The elements of $M^{(j)}$ and $D^{(j)}$ are then

$$B_j^{(j)} = B_{j+1}^{(j)} = [(1/2 - \lambda)\ (C - \theta F) + \theta K]^{(j)}, \qquad \textbf{(3-25)}_1$$

(This is true only because the diagonal terms in the element capacitance, conductance, and generation matrices, are the same.)

$$-A_{j+1}^{(j)} = [\lambda(C - \theta F) - \theta K]^{(j)}, \qquad \textbf{(3-25)}_2$$

$$D_j^{(j)} = [(1/2 - \lambda)\ (C - \theta F) - (1 - \theta)K]^{(j)} T_j^n$$

$$+[\lambda(C - \theta F) + (1 - \theta)K]^{(j)}\ T_{j+1}^n + \frac{[\mathcal{V}\dot{Q}(\overline{T}^n)]^{(j)}}{2}\ , \qquad \textbf{(3-25)}_3$$

and

$$D_{j+1}^{(j)} = [\lambda(C - \theta F) + (1 - \theta)K]^{(j)}\ T_j^n$$

$$+\ [(1/2 - \lambda)(C - \theta F) - (1 - \theta)K]^{(j)}\ T_{j+1}^n$$

$$+\ \frac{[\mathcal{V}\dot{Q}(\overline{T}^n)]^{(j)}}{2}\ . \qquad \textbf{(3-25)}_4$$

The assembly algorithm for the global matrix is then given as:

1) Define three vectors, A, B, D, of length NTN, the number of nodes in the problem.
2) Initialize A, B, D to zero.
3) For the *j-th element*, compute *element* properties k, ρ, C_p, A, $\mathcal{V}$, $\dot{Q}$, etc.
4) Compute the components of the element matrices, $M^{(j)}$, $D^{(j)}$ from Eq. (3-25).
5) Assemble these components with those from earlier elements summing only those values of −A, B, and D with the *same node number*. Typical FORTRAN statements are, for example,

   B(5) = B(5) + (EQN. $(3\text{-}25)_1$)
   D(5) = D(5) + (EQN. $(3\text{-}25)_{3\text{ or }4}$)

where the left side represents, after all elements have been scanned, the assembled components of the global matrices.

6) Examine the current element number index. If the last element contribution has been added, go to Step 7. If not, go to Step 3.
7) Sum, to the first and last element As, Bs, and Ds, the boundary condition contributions.
8) The assembly process for the global matrices is completed.

This algorithm is implemented within SODDIT as SUBROUTINEs FORMM (for FORM Matrix), FORMSM (for FORM Symmetric Matrix), and BC (Boundary Condition).

Symmetry

The element "stiffness" matrix, Eq. (3-24), was presumed to be symmetric. It is essential that any modification to SODDIT retain this property. This is necessary because the tridiagonal matrix equation solver uses a modified form of the Thomas algorithm for symmetric equations. At this point, it is necessary to examine the circumstances under which symmetry could be lost. Since it is the A-contributions that are the off-diagonal elements, symmetry will be determined by them.

From Eq. $(3\text{-}25)_2$, the components of the A-coefficients are the capacitance and conductance of the element, C and K, and the presence of a temperature-dependent energy source within the element, F. These are defined as

$$\begin{aligned} C^{(j)} &= (\rho C_p \mathcal{V})^{(j)}/\Delta t \\ K^{(j)} &= (kA/\ell)^{(j)} \\ F^{(j)} &= [\mathcal{V}(d\dot{Q}/dT)_n]^{(j)} \ . \end{aligned} \qquad \textbf{(3-26)}$$

Central to the development of the difference equations was the assumption that each element had associated with it a uniform density (ρ), specific heat (C_p), thermal conductivity (k), energy generation function [$\dot{Q}(T)$], volume ($\mathcal{V}$), and cross-sectional area (A). With this true, all such properties can be moved outside the control volume integrals. A consequence is that the element and global-stiffness matrices must be symmetric. If, however, any one of the properties is not uniform within an element, symmetry is destroyed.

As an example, in deriving the difference form of the energy-generation-rate effect on the general problem [when $\dot{Q} = \dot{Q}(T)$], the question arises: at what temperature within an element should $d\dot{Q}/dT$ be evaluated? [Refer to the integrals immediately following Eq. (3-9).] If it is evaluated at the weighted average temperature associated with one-half of a control volume [e.g., $(3T_j + T_{j+1})/4$ for the right half of a control volume and $(T_{j-1} + 3T_j)/4$ for the left half], then within an element $d\dot{Q}/dT$ will have *two* values. The consequence is assymmetry. The solution is to evaluate $d\dot{Q}/dT$ at the element temperature, $(T_j + T_{j+1})/2$.

It is interesting to notice that boundary conditions do not influence symmetry. This is evident from examining Eqs. (3-12, -14, -15) in that only the Bs and Ds for elements 1 and NTE are effected. Neither does any energy-generation-rate function that is independent of temperature effect symmetry, since then $d\dot{Q}/dT \equiv 0$.

In summary, the element matrices representing conduction, capacitance, and energy-generation effects can be written as follows:

Element Conduction—

$$\left(\frac{kA}{\Delta x}\right)^{(j)} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} T_j \\ T_{j+1} \end{Bmatrix} \qquad \textbf{(3-27)}_1$$

Element Capacitance—

$$(\rho C \mathcal{V})^{(j)} \begin{bmatrix} 1/2 - \lambda & \lambda \\ \lambda & 1/2 - \lambda \end{bmatrix} \begin{Bmatrix} \dot{T}_j \\ \dot{T}_{j+1} \end{Bmatrix} \qquad \textbf{(3-27)}_2$$

Element Energy Generation—

$$(\dot{Q}^n \mathcal{V}^{(j)} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \left(\mathcal{V}\frac{d\dot{Q}}{dT}\right)^{(j)} \begin{bmatrix} 1/2 - \lambda & \lambda \\ \lambda & 1/2 - \lambda \end{bmatrix} \times \begin{Bmatrix} T_j^{n+1} - T_j^n \\ T_{j+1}^{n+1} - T_{j+1}^n \end{Bmatrix} . \qquad \textbf{(3-27)}_3$$

Within SODDIT, the global equations are assembled on an element-by-element basis.

4. Output From SODDIT

Overview

The printed output from SODDIT is divided into three main sections. First, SUBROUTINE INPOUT prints the majority of the input information. Second, complete temperature-profile and surface-energy-balance information are printed at all times designated as print times on the input, using SUBROUTINE OUTPUT. Finally, summary information is written for temperature as a function of time for selected depths (both direct and inverse problems), heat flux as a function of time, and for inverse problems only, a comparison between experimental and computed temperatures. Sample output for both a direct and an inverse problem are included in Appendixes A and B, respectively.

Printing Of Input Information

The majority of the input information is printed to verify that all the input information is correct. Three records of Title information are printed first, then all 80 of the control flags KR(I) along with their appropriate columns are printed. Next, the times when the print intervals change and the corresponding print intervals are listed; the first print time is the initial problem time and the final (nonzero) print time is the final problem time. A zero value of either print time or print interval simply means that the input value was a blank field. The depths at which temperature histories are determined are printed next; if KR(24) = 0, then these depths are automatically set to the front and back face along with all the material interface values. The general problem constants are printed next, followed by the material property data; density (ρ), and the table $C_p(T)$, k(T), and $\epsilon(T)$. The only noninput quantity is the ENTHALPY, which is determined by integrating the temperature-dependent specific heat from the reference temperature TREF up to the tabular temperature entries and adding the heat of formation DHF (for nonablating problems, this quantity is not used). Next, the nodal data of node number, node position relative to node 1 (the front face), and the initial temperature are printed [the units of the initial temperature printed will depend on KR(19)]. The element data of element number (NE); the two node numbers associated with the element (NI and NJ); material number (NMAT); thickness of the element (DXI) in input units (not necessarily consistent units); cross-sectional area of the element (AREAI) in input units; volume of element (VOLI) in input units; and a flag (NGAP) indicating presence or absence of a gap appear next. If energy sources are present, the constants used in computing the generation rate (QA, QB, QC, QD, QE) are printed along with the material number for the section of the domain experiencing generation (MATGEN).

Some comments about AREAI are appropriate: for a planar geometry, AREAI = 1.0; for a cylindrical geometry, the quantity labeled AREAI is actually area/Z where Z is the length of the cylinder. Since this scale factor Z cancels from the difference equations, it was omitted from the code.

The time-dependent boundary condition data of input Block 7 is printed next; the "1" in the first column indicates front face while the "N" indicates back face. The remaining output of input information depends on whether a direct or an inverse problem is being solved. For inverse problems (IBCTY1 = 4), the experimental temperature data as a function of time are printed as the "front-face" boundary condition; the second boundary condition table is the "back-face" boundary condition. The output quantities are the same as the input, with the exception of P, which is a pressure that is used only for Option 1 problems. For direct problems, both the "front-" and "back-face" boundary condition tables have the same appearance.

Front-Face Energy Balance And Temperature Profile Information

Corresponding to the print times established in Block 3 of the input, information relating to the front-face energy balance (surface energy balance at node 1) and the complete temperature profile are printed with the same three records of title information as a heading followed by the date and time execution was initiated. The value of the time-differencing parameter THETA and flag KR(12) that controls the energy-balance weighting factor are then printed; this is useful information to have in case the output temperature profile is separated from the output of input information. The problem time (in consistent units) is printed next. The code determines the problem time step based on several internal criteria and this value as well as the cumulative number of time steps required to reach the print time are printed.

The front-face surface-energy-balance terms are printed next; if ITER = 0, the surface energy balance has been linearized and no iterations were performed. If ITER > 0, then a Type 1 surface energy balance has been used that requires iteration on the unknown surface temperature through the nonlinear energy balance. Refer to Section 3, Nodal Equations for Derivative Boundary Conditions. The surface energy balance given by Eq. (3-12) contains a weighted average of heat flux events at time steps n and n + 1. They are defined as follows:

$$\text{QDIFF} = h_o(t)[T_{\infty_o}(t) - T(0,t)]\,\theta + h_o\,(t - \Delta t) \times [T_{\infty_o}(t - \Delta t) - T(0,t - \Delta t)](1 - \theta) \quad \textbf{(4-1)}$$

$$\text{QRIN} = \alpha_w(t)q''_{rad_o}(t)\theta + \alpha_w\,(t - \Delta t) \times q''_{rad_o}(t - \Delta t)(1 - \theta) \quad \textbf{(4-2)}$$

$$\text{QROUT} = \epsilon_w(t)[T^4(0,t) - T^4_{rad_o}(t)]\theta + \epsilon_w\,(t - \Delta t) \times [T^4(0,t - \Delta t) - T^4_{rad_o}(t - \Delta t)]1 - \theta) \; . \quad \textbf{(4-3)}$$

$$\text{QCHEM} = \begin{cases} 0. \text{ for non-ablating problems} \\ \rho\dot{s}Q^* \text{ for } Q^* \text{ option ablating problems} \\ \rho\dot{s}(i_w - i_c) \text{ for thermochemical ablation} \end{cases} \quad \textbf{(4-4)}$$

These quantities are effective averages at time = $\theta t + (1 - \theta)(t - \Delta t)$. It is convenient to define a net heat flux as

$$\text{QNET} = \text{QDIFF} + \text{QRIN} - \text{QROUT} - \text{QCHEM} \quad \textbf{(4-5)}$$

where QNET physically represents the net heat flux available to be conducted into the body though node 1. If the surface energy balance is linear in the unknown surface temperature, then iteration is not required. The approach taken in the SODDIT code for IBCTY1 = 3 is to linearize the nonlinearities through a Taylor series expansion; the validity of this approximation can then be ascertained by observing the difference between QCOND and QNET as printed.

To fully understand the QCOND term and how it differs from QNET, it will be beneficial to understand the Thomas algorithm for solving the tridiagonal system of equations. The difference equations can be written as

$$b_1T_1 - a_2T_2 = d_1 \quad \textbf{(4-6)}_1$$

$$-a_jT_{j-1} + b_jT_j - a_{j+1}T_{j+1} = d_j,\ j = 2, 3, \ldots, N-1 \quad \textbf{(4-6)}_2$$

$$-a_NT_{N-1} + b_NT_N = d_N \quad \textbf{(4-6)}_3$$

where the time superscript has been omitted. The variant of the Thomas algorithm used in SODDIT starts at the back-face node N and works toward the front-face node 1. The procedure is

Forward elimination—

$$\omega_N = 1/b_N,\ e_N = a_N\omega_N,\ f_N = d_N\omega_N \quad \textbf{(4-7)}_1$$

$$\left.\begin{aligned} \omega_j &= 1/(b_j - a_{j+1}e_{j+1}) \\ e_j &= a_j\omega_j \\ \gamma_j &= d_j + a_{j+1}f_{j+1} \\ f_j &= \gamma_j\omega_j \end{aligned}\right\} j = N-1, N-2, \ldots, 1 \quad \textbf{(4-7)}_2$$

Back substitution—

$$T_1 = f_1 = \gamma_1\,\omega_1 \quad \textbf{(4-8)}_1$$

$$T_j = e_jd_{j-1} + f_j, \qquad j = 2,3,\ldots,N \; . \quad \textbf{(4-8)}_2$$

The front-face surface energy balance, as given by Eq. (3-12), can be written in symbolic form as

$$b_1' T_1^{n+1} - a_2' T_2^{n+1} = d_1' + A_1 q''(T_1^{n+1}) \quad (4\text{-}9)$$

where

$$q''(T_1^{n+1}) = \theta q_0''^{n+1}(T_1^{n+1}) + (1-\theta) q_0''^{n}(T_1^{n}) \; . \quad (4\text{-}10)$$

The superscript "′" on the b_1', a_2', and d_1' terms is used to distinguish them from the terms in Eq. (3-22). Compare Eq. (4-6), with Eq. (4-9); Eq. (4-6) contains the linearization for $q''(T_1^{n+1})$.

The solution procedure is to use the linearized form of the surface energy balance and then calculate the surface temperature from Eq. (4-8). Using the front-face temperature predicted from the linearized energy balance, the surface-energy-balance terms QDIFF, QRIN, QROUT, and QNET can be calculated. Based upon this surface temperature, Eq. (4-9) could be solved for the net heat flux that satisfies the energy balance and the surface temperature; this heat flux will be denoted by q_{cond}''. From Eqs. (4-7) and (4-8), the surface temperature can be written as

$$T_1^{n+1} = \gamma_1' \omega_1' = \omega_1' [d_1' + q_{cond}'' A_1 + a_2' f_2] \quad (4\text{-}11)$$

where

$$\omega_1' = b_1' - a_2' e_2, \; \gamma_1' = d_1' + a_2' f_2 \; . \quad (4\text{-}12)$$

It is not necessary to put the superscript on e_2 and f_2 since they are independent of the front-face boundary conditions. Solving Eq. (4-11) for q''_{cond} gives

$$q_{cond}'' = \frac{T_1^{n+1}/\omega_1' - \gamma_1'}{A_1} = \text{QCOND} \; . \quad (4\text{-}13)$$

For a linear front-face boundary condition, QCOND = QNET. For nonlinear problems, the discrepancy between QCOND and QNET will be an indicator of the validity of the linearization process. As the time step becomes smaller, then the surface-energy-balance error can be made arbitrarily small.

In the case where IBCTY1 = 1, the energy balance at the surface is nonlinear and iteration is required to solve for the unknown surface temperature that satisfies energy conservation at the boundary. The process used in the iteration is completely discussed in Section 3, Nodal Equations for Derivative Boundary Conditions. Being an iterative solution method, ITER is simply the number of iterations needed in the Newton's method to produce the wall temperature. Typically, only two or three iterations are required. The remaining quantities printed at each print time, t, are interpreted as follows:

$$\text{QCOND} = [T_1(t)/\omega'_1 - \gamma'_1]/A_1 \quad (4\text{-}14)$$

$$\text{QDIFF} = \theta C_h(t)[i_r(t) - i_w(t)] + (1-\theta) C_h(t-\Delta t)[i_r(t-\Delta t) - i_w(t-\Delta t)] \quad (4\text{-}15)$$

QRIN = Equation (4-2)

QROUT = Equation (4-3)

$$\text{QNET} = \text{QDIFF} + \text{QRIN} - \text{QROUT} \quad (4\text{-}16)$$

and

$$\text{QCHEM} = \begin{cases} 0 \text{ for nonablating problems.} \\ (\dot{m}_c'' Q^*)^{n+1} \theta + (\dot{m}_c'' Q^*)^n (1-\theta) \end{cases}$$

In each case, the fluxes are weighted between time steps n and n + 1, and when $T_1(t)$ is used, it is the value after iteration has converged.

In the next two lines of surface-energy-balance information, several of the items are applicable only for ablation problems; however, all items are defined here for completeness:

BP = B′ = $(\rho \dot{s})/(\rho_e U_e St)$ = ablation mass flux-per-unit-area normalized by boundary layer edge mass flux-per-unit-area times heat-transfer Stanton number

MDOTC = $\dot{m}_c''$ = $\rho \dot{s}$ = ablation mass flux-per-unit-area

SDOT = $\dot{s}$ = surface recession rate

STOT = S = total surface recession

PRESSURE = P = pressure (used only for surface energy balance Type 1)

TSG = T_{s_g} = guessed value of surface temperature based on Taylor series expansion in time from two previous time steps

HC = i_c = enthalphy of ablation material at the wall temperature

HW = i_w = enthalpy of gas adjacent to ablating surface, at the surface temperature

$$HR = \begin{cases} i_r & = \text{recovery enthalpy of free stream for Option 1} \\ T_w & = \text{specified temperature for Option 2} \\ T_\infty & = \text{temperature of free stream for Option 3} \end{cases}$$

$$CH = \begin{cases} St & = \text{heat-transfer Stanton number for Option 1} \\ h & = \text{convective heat-transfer coefficient for Option 3} \end{cases}$$

$CHO = St_0$ = heat-transfer Stanton number for zero blowing

CH/CHO = ratio of actual Stanton number to unblown Stanton number (always unity for nonablating problems)

The complete temperature profile is printed next for each node number, NODE; MATL is the material number on the left side of the node; MATR is the material number on the right side of the node; DEPTH is the distance of the node from the front face (node 1); and TEMPERATURE is the node temperature in consistent units. When MATL $\neq$ MATR, then this node is located at a material interface.

The user should note that when cylindrical or spherical geometry is used, a *positive* value of RADIUS means that node 1 is on the *outside* and distance (depth) is measured inward from x = 0 at node 1; a *negative* value of RADIUS means that node 1 is on the *inside* surface and distance (depth) is then measured outward from node 1 on the inside surface (x = 0).

Summary Information

For inverse problems (IBCTY1 = 4), a summary of calculated surface temperature, calculated heat flux, and a comparison between input and calculated temperature at the sensor location is presented for each calculation time; the TIME column is the problem time. When some of the temperature information is skipped (ISKP>1), only those times for which calculations were performed will be printed. The TSURF column is the calculated surface temperature; Q is the calculated heat flux in consistent units. TINPUT and TCALC are the experimental input temperature and calculated temperature, respectively, for the first sensor; these two columns will be repeated for each subsequent sensor, up to a maximum of seven sensors. Note that for inverse problems, the final calculation time may not be the same as the final time for which experimental data are available because of the number of future times (NFT); if N is the index number of the last experimental temperature measurement, then the index number of the last calculation time will be N-(NFT-1).

For both inverse and direct problems, the temperature history of selected "sensors" will be printed. If KR(24) = 0, then the front- and back-face temperatures along with material interfaces will automatically be printed. If KR(24) $\neq$ 0, then depths at which the temperature is to be monitored must be input; these depths should lie within the problem domain since linear interpolation is used. The line beginning with DEPTH gives the distances from the front face (x=0) for which temperature histories will be printed. Next, the print times (TIME) and temperatures in consistent units are printed; the final line of output is the time the computer used to execute the FORTRAN statements (i.e., the CPU time) in seconds. If there is more than one problem to be computed, the CPU time is for each problem, individually.

5. Numerical Examples

Introduction

The object of this section is to present results obtained from SODDIT for various numerical test problems. Both direct and inverse heat conduction problems are included. In all cases, the results generated using SODDIT are compared to other data, either exact solutions or other numerically generated data. The intent is not to present an exhaustive evaluation of the program, but rather to provide benchmark data for the user. Such data are useful for new users of the program and in debugging the program. The input files used to generate the output from SODDIT are included as a part of the tables that follow each problem below. The first series of test problems are direct; the section closes with a set of inverse problems.

Direct Heat Conduction Problems

Plane Wall with Energy Generation

Consider a very large plane wall of width 2a. The wall material generates a uniform rate of energy per unit volume, A_0, that begins at $t = 0$. Initially, the wall is at a uniform temperature, T_0 and as time passes the faces of the wall are held at T_0. Assuming constant material properties, the problem is governed by

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{\alpha}\frac{\partial T}{\partial t} = -\frac{A_0}{k};\ -a < x < a,\ t > 0 \qquad (5\text{-}1)_1$$

with

$$T(x,0) = T_0,\ -a \le x \le a \qquad (5\text{-}1)_2$$

$$T(\pm a,t) = T_0,\ t > 0 \qquad (5\text{-}1)_3$$

$$T_x(0,t) = 0,\ t > 0 \qquad (5\text{-}1)_4$$

If $a = \alpha = A_0 = k = 1$ and $T_0 = 0$, the exact solution (Reference 3, p. 130) can be given as

$$T(x,t) = \frac{(1-x^2)}{2} - 16\sum_{n=0}^{\infty}\frac{(-1)^n}{b_n^3}\cos(b_n x/2)\exp(-b_n^2 t/4) \qquad (5\text{-}2)_1$$

where $b_n = (2n+1)\pi$ $(5\text{-}2)_2$

are the eigenvalues and

$$q_s'(t) = 1 - 8\sum_{n=0}^{\infty}\frac{(-1)^n}{b_n^2}\sin(b_n/2)\exp(-b_n^2 t/4)\ . \qquad (5\text{-}3)$$

$q_s'(t)$ is the heat transfer rate across A, the surface area of the wall, defined by

$$q_s'(t) = -k\frac{\partial T}{\partial x}\bigg|_a\ . \qquad (5\text{-}4)$$

Table 5 displays the results of the computations for this problem. The negative sign on SODDIT output simply means that heat is flowing from node 2 to node 1.

Infinite Solid Cylinder with Energy Generation

Consider heat conduction in a very large circular cylinder of radius, a, that initially is at a uniform temperature, T_0. At $t = 0$, a uniform volumetric rate of energy generation exists within the cylinder of magnitude A_0, while its surface is held at T_0. This problem is governed by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) - \frac{1}{\alpha}\frac{\partial T}{\partial t} = -\frac{A_0}{k}\ 0 < r < a,\ t > 0 \qquad (5\text{-}5)_1$$

with

$$T(r,0) = T_0,\ \ 0 \le r \le a \qquad (5\text{-}5)_2$$

$$T(a,t) = T_0,\ \ t > 0 \qquad (5\text{-}5)_3$$

$$T_r(0,t) = 0,\ \ t > 0 \qquad (5\text{-}5)_4$$

Table 5. Results for a Plane Wall with Energy Sources and the Input Data File Used by SODDIT to Compute the Results. ($\Delta x = .05$, $\Delta t = .001$, fully implicit time integration with FCV method.)

	t = 0.1		t = 1.0	
	Exact	SODDIT	Exact	SODDIT
$q_s''(t)$	.3568234	-.33222	.9312597	-.90616
Depth	T	T	T	T
.0000	.00000	3.56657E-21	.00000	9.31112E-21
.0500	.01663	1.66108E-02	.04532	4.53082E-02
.1000	.03098	3.09445E-02	.08815	8.81377E-02
.1500	.04327	4.32213E-02	.12853	1.28509E-01
.2000	.05372	5.36558E-02	.16648	1.66444E-01
.2500	.06253	6.24548E-02	.20200	2.01963E-01
.3000	.06990	6.98146E-02	.23513	2.35085E-01
.3500	.07601	7.59198E-02	.26588	2.65830E-01
.4000	.08104	8.09411E-02	.29428	2.94216E-01
.4500	.08513	8.50351E-02	.32033	3.20261E-01
.5000	.08844	8.83430E-02	.34406	3.43981E-01
.5500	.09108	9.09905E-02	.36547	3.65393E-01
.6000	.09318	9.30884E-02	.38460	3.84511E-01
.6500	.09482	9.47325E-02	.40144	4.01347E-01
.7000	.09608	9.60043E-02	.41601	4.15914E-01
.7500	.09705	9.69721E-02	.42832	4.28222E-01
.8000	.09776	9.76919E-02	.43838	4.38280E-01
.8500	.09827	9.82076E-02	.44620	4.46095E-01
.9000	.09861	9.85528E-02	.45178	4.51673E-01
.9500	.09881	9.87508E-02	.45512	4.55018E-01
1.0000	.09887	9.88152E-02	.45624	4.56133E-01

```
000000000 310000000 0000000001000000000 000000000 000000000 000000000 0000000000
      PLANE WALL INITIALLY AT UNIFORM TEMPERATURE
      FIXED SURFACE TEMPERATURE AT X = 0 AND INSULATED AT X = L
      UNIFORM ENERGY GENERATION
        0.        1.
        .1
      .001      .001        1.
        1.
 0       0.        1.        1.
 1       1.        1.        1.
        0.
    1   20    1.0
0
        0.        0.    1    2
        0.        0.    2    3
 0      0.        0.
-1     10.        0.
 0      0.        0.        0.        0.
+1     10.        0.        0.        0.
1.0         0.0       0.0       0.0       0.0         1
```

If $a = \alpha = k = A_0 = 1$ and $T_0 = 0$, the exact solution (Reference 3, p. 204) becomes

$$T(r,t) = \frac{(1-r^2)}{4} - 2\sum_{n=1}^{\infty} \frac{\exp(-b_n t)\, J_0(b_n r)}{b_n^3 J_1(b_n)} \qquad (5\text{-}6)_1$$

where the eigenvalues b_n are determined by the eigencondition

$$J_0(b_n) = 0\ . \qquad (5\text{-}6)_2$$

The rate of energy transfer at the cylinder's surface is

$$q_s''(t) = \frac{1}{2} - 2\sum_{n=1}^{\infty} \frac{\exp(-b_n t)}{b_n^2}\ . \qquad (5\text{-}7)$$

These results and those of SODDIT using the same values for the coefficients are given in Table 6.

Solid Sphere with Energy Generation

A solid sphere of radius a is initially at a uniform temperature T_0. At $t = 0$, a uniform rate of energy generation of magnitude A_0 begins while the surface temperature is held at T_0. The diffusion of energy within the solid is governed by

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) - \frac{1}{\alpha}\frac{\partial T}{\partial t} = -\frac{A_0}{k} \quad 0 < r < a,\ t > 0 \qquad (5\text{-}8)_1$$

with

$$T(r,0) = T_0, \quad 0 \le r \le a \qquad (5\text{-}8)_2$$

and

$$T(a,t) = T_0, \quad t > 0 \qquad (5\text{-}8)_3$$

$$T_r(0,t) = 0, \quad t > 0. \qquad (5\text{-}8)_4$$

If $a = A_0 = \alpha = k = 1$ and $T_0 = 0$, the exact solution (Reference 3, p. 243) to problem (5-8) is

$$T(r,t) = \frac{(1-r^2)}{6} + \frac{2}{r}\sum_{n=1}^{\infty} \frac{(-1)^n}{b_n^3} \sin(b_n r)\exp(-b_n^2 t) \qquad (5\text{-}9)_1$$

where the eigenvalues are

$$b_n = n\pi. \qquad (5\text{-}9)_2$$

The surface heat transfer rate is then

$$q_s''(t) = \frac{1}{3} - 2\sum_{n=1}^{\infty} \frac{\exp(-b_n^2 t)}{b_n^2}\ . \qquad (5\text{-}10)$$

These results and those of SODDIT using the identical values for the coefficients are given in Table 7.

Plane Wall with Energy Generation and Convection

The plane wall described for Eq. (5-1) is now experiencing convection heat transfer at its surfaces, so the condition $(5\text{-}1)_3$ is changed to

$$k\frac{\partial T(a,t)}{\partial x} = -h[T(a,t) - T_\infty], \quad t > 0. \qquad (5\text{-}11)$$

If again, $a = \alpha = A_0 = k = 1$ and $h = 1$ while $T_\infty = 0$, then the exact solution (Reference 3, p. 132) can be given as

$$T(x,t) = \frac{1}{2}\left\{2 + (1-x^2) - 4\sum_{n=1}^{\infty} \frac{\cos(b_n x)\exp(-b_n^2 t)}{b_n^2[b_n^2 + 2]\cos b_n}\right\} \qquad (5\text{-}12)_1$$

where the eigenvalues are the roots of

$$b_n \tan b_n = 1. \qquad (5\text{-}12)_2$$

The surface heat-transfer rate is

$$q_s''(t) = 1 - 2\sum_{n=1}^{\infty} \frac{\exp(-b_n^2 t)}{b_n^2(b_n^2 + 2)}\ . \qquad (5\text{-}13)$$

These results and those for SODDIT using the same coefficient values are given in Table 8.

Table 6. Results for an Infinite Cylinder with Energy Sources and the Input Data File Used by SODDIT to Compute the Results. ($\Delta r = .05$, $\Delta t = .001$, fully implicit time integration with FCV method.)

	t = 0.1		t = 1.0	
	Exact	SODDIT	Exact	SODDIT
$\ddot{q}_s(t)$	.3029121	-.28539	.4989352	-.48655
Depth	T	T	T	T
.0000	.00000	1.8996793E-20	.00000	3.1347482E-20
.0500	.01429	1.4269650E-02	.02432	2.4327268E-02
.1000	.02695	2.6903658E-02	.04739	4.7402782E-02
.1500	.03808	3.8014992E-02	.06921	6.9227156E-02
.2000	.04781	4.7720589E-02	.08977	8.9801037E-02
.2500	.05624	5.6139323E-02	.10909	1.0912510E-01
.3000	.06351	6.3389995E-02	.12715	1.2720005E-01
.3500	.06972	6.9589431E-02	.14397	1.4402661E-01
.4000	.07500	7.4850739E-02	.15954	1.5960553E-01
.4500	.07944	7.9281778E-02	.17386	1.7393758E-01
.5000	.08315	8.2983862E-02	.18693	1.8702355E-01
.5500	.08622	8.6050737E-02	.19875	1.9886429E-01
.6000	.08873	8.8567809E-02	.20933	2.0946070E-01
.6500	.09078	9.0611623E-02	.21867	2.1881379E-01
.7000	.09241	9.2249577E-02	.22675	2.2692475E-01
.7500	.09370	9.3539822E-02	.23360	2.3379515E-01
.8000	.09469	9.4531342E-02	.23920	2.3942724E-01
.8500	.09542	9.5264208E-02	.24355	2.4382485E-01
.9000	.09591	9.5770140E-02	.24666	2.4699583E-01
.9500	.09620	9.6074276E-02	.24853	2.4896232E-01
1.0000	.09630	9.6207774E-02	.24915	2.4984173E-01

```
000000000 310000000 0000000001000000000 000000000 000000000 000000000 0000000000
      SOLID CIRCULAR CYLINDER INITIALLY AT UNIFORM TEMPERATURE
      FIXED SURFACE TEMPERATURE
      UNIFORM ENERGY GENERATION
        0.        1.
        .1
      .001      .001        1.        1.        0.        1.        1.
        1.
 0        0.        1.        1.        0.
 1       10.        1.        1.        0.
        0.
    1   20  1.0
0
        0.        0.    1    2
        0.        0.    2    3
 0      0.        0.
-1     10.        0.
 0      0.        0.        0.        0.
+1     10.        0.        0.        0.
      1.    0.0       0.0       0.0       0.0          1
```

Table 7. Results for a Solid Sphere with Energy Generation and the Input Data File Used by SODDIT to Compute the Results. ($\Delta r = .05$, $\Delta t = .001$, fully implicit time integration with FCV method.)

	t = 0.1		t = 1.0	
	Exact	SODDIT	Exact	SODDIT
$q_s''(t)$	.25683	-.24440	.33332	-.32520
Depth	T	T	T	T
.0000	.00000	3.2190194E-21	.00000	4.1886490E-20
.0500	.01224	1.2219931E-02	.01625	1.6260088E-02
.1000	.02331	2.3267938E-02	.03167	3.1687370E-02
.1500	.03326	3.3196345E-02	.04625	4.6281917E-02
.2000	.04215	4.2063700E-02	.06000	6.0043814E-02
.2500	.05004	4.9933540E-02	.07291	7.2973165E-02
.3000	.05700	5.6873071E-02	.08500	8.5070088E-02
.3500	.06310	6.2951779E-02	.09625	9.6334730E-02
.4000	.06841	6.8240050E-02	.10666	1.0676727E-01
.4500	.07299	7.2807822E-02	.11624	1.1636793E-01
.5000	.07692	7.6723336E-02	.12499	1.2513700E-01
.5500	.08026	8.0051994E-02	.13291	1.3307484E-01
.6000	.08307	8.2855366E-02	.13999	1.4018194E-01
.6500	.08541	8.5190356E-02	.14624	1.4645899E-01
.7000	.08733	8.7108532E-02	.15166	1.5190698E-01
.7500	.08887	8.8655637E-02	.15624	1.5652741E-01
.8000	.09008	8.9871302E-02	.15999	1.6032275E-01
.8500	.09099	9.0789044E-02	.16291	1.6329744E-01
.9000	.09162	9.1436894E-02	.16499	1.6546075E-01
.9500	.09199	9.1840104E-02	.16624	1.6683640E-01
1.0000	.09211	9.2031308E-02	.16666	1.6749774E-01

```
000000000 310000000 0000000001000000000 000000000 000000000 000000000 0000000000
      SOLID SPHERE INITIALLY AT UNIFORM TEMPERATURE
      FIXED SURFACE TEMPERATURE
      UNIFORM ENERGY GENERATION
        0.        1.
        .1
      .001      .001        1.        1.        0.        1.        2.
        1.
 0        0.        1.        1.
 1        1.        1.        1.
        0.
    1   20  1.0
0
        0         0.    1    2
        0.        0.    2    3
 0      0.        0.
-1     10.        0.
 0      0.        0.        0.        0.
+1     10.        0.        0.        0.
   1.0      0.0      0.0       0.0       0.0         1
```

Table 8. Results for an Infinite Plane Wall with Energy Generation and Convection and the Input Data File Used by SODDIT to Compute the Results. ($\Delta x = .05$, $\Delta t = .001$, fully implicit time integration with FCV method.)

	$t'' = 0.1$		$t = 1.0$	
	Exact	SODDIT	Exact	SODDIT
$q_s''(t)$	.08040325	-.080383	.5296028	-.52953
Depth	T	T	T	T
.0000	.08040	8.03834E-02	.52960	5.29531E-01
.0500	.08409	8.40685E-02	.55528	5.55198E-01
.1000	.08718	8.71501E-02	.57936	5.79279E-01
.1500	.08974	8.97082E-02	.60190	6.01813E-01
.2000	.09185	9.18161E-02	.62293	6.22841E-01
.2500	.09357	9.35398E-02	.64250	6.42400E-01
.3000	.09497	9.49383E-02	.66063	6.60526E-01
.3500	.09609	9.60640E-02	.67736	6.77251E-01
.4000	.09699	9.69628E-02	.69272	6.92608E-01
.4500	.09770	9.76745E-02	.70674	7.06626E-01
.5000	.09826	9.82332E-02	.71945	7.19331E-01
.5500	.09869	9.86680E-02	.73087	7.30750E-01
.6000	.09902	9.90032E-02	.74103	7.40905E-01
.6500	.09928	9.92589E-02	.74994	7.49816E-01
.7000	.09947	9.94518E-02	.75763	7.57502E-01
.7500	.09961	9.95952E-02	.76411	7.66979E-01
.8000	.09971	9.96995E-02	.76939	7.69259E-01
.8500	.09978	9.97728E-02	.77348	7.73355E-01
.9000	.09983	9.98211E-02	.77640	7.76274E-01
.9500	.09986	9.98485E-02	.77815	7.78023E-01
1.0000	.09987	9.98574E-02	.77874	7.78606E-01

```
000000000 310000000 0000000001000000000 000000000 000000000 000000000 0000000000
      PLANE WALL INITIALLY AT UNIFORM TEMPERATURE
      CONVECTION AT THE SURFACE X = 0 AND INSULATED AT X = L
      UNIFORM ENERGY GENERATION
        0.        1.
        .1
      .001      .001        1.        1.        0.
        1.
 0       0.        1.        1.        0.
 1      10.        1.        1.        0.
        0.
    1   20  1.0
0
        0.        0.    1   3
        0.        0.    2   3
 0      0.        0.        0.        1.
-1     10.        0.        0.        1.
 0      0.        0.        0.        0.
+1     10.        0.        0.        0.
     1.0     0.0       0.0       0.0       0.0         1
```

Infinite Solid Cylinder with Energy Generation and Convection

The circular cylinder described for Eq. (5-5) is now experiencing convection heat transfer to the surrounding fluid so that condition Eq. $(5\text{-}5)_4$ is changed to

$$k\frac{\partial T(a,t)}{\partial r} = -h[T(a,t) - T_\infty], \quad t > 0 \ . \tag{5-14}$$

If again, $a = \alpha = A_0 = k = 1$ and $h = 1$ while $T_\infty = 0$, then the exact solution for T(r,t) (Reference 3, p. 205) can be written as

$$T(r,t) = \frac{(1-r^2)}{4} + \frac{1}{2} - 2\sum_{n=1}^{\infty} \frac{\exp(-b_n^2 t)\, J_0(b_n r)}{b_n^2 [b_n^2 + 1]\, J_0(b_n)} \tag{5-15}_1$$

where the eigenvalues are given as the roots of

$$b_n J_1(b_n) = J_0(b_n) \tag{5-15}_2$$

The surface heat-transfer rate is then

$$q_s''(t) = \frac{1}{2} - 2\sum_{n=1}^{\infty} \frac{\exp(-b_n^2 t)\, J_1(b_n)}{b_n [b_n^2 + 1]\, J_0(b_n)} \ . \tag{5-16}$$

These results and those for SODDIT using the same values for the coefficients are given in Table 9.

Solid Sphere with Energy Generation and Convection

The solid sphere described for Eq. (5-8) is now experiencing convection heat transfer to the surrounding fluid so that condition Eq. $(5\text{-}8)_4$ is changed to

$$k\frac{\partial T(a,t)}{\partial r} = -h[T(a,t) - T_\infty] \ . \tag{5-17}$$

If again, $a = \alpha = A_0 = k = 1$ and $h = 1$ while $T_\infty = 0$, then the exact solution for T(r,t) (Reference 3, p. 245) can be written as

$$T(r,t) = \frac{(1-r^2)}{6} + \frac{1}{3} - \frac{2}{r}\sum_{n=1}^{\infty} \frac{\sin(b_n r)\exp(-b_n^2 t)}{b_n^4 \sin(b_n)} \tag{5-18}_1$$

where the eigenvalues b_n satisfy the eigencondition

$$\cot(b_n) = 0 \ . \tag{5-18}_2$$

The surface energy flux from the sphere is then

$$q_s''(t) = \frac{1}{3} - 2\sum_{n=1}^{\infty} \frac{\exp(-b_n^2 t)}{b_n^4} \ . \tag{5-19}$$

These results and those for SODDIT using the same values for the coefficients are given in Table 10.

Three-Layer Cylinder

As an example of the use of SODDIT on a composite material problem, consider the transient thermal diffusion in a long cylindrical shell composed of three different materials as shown in Figure 9. Initially, the composite shell has a uniform temperature, T_i = 536R, but at t = 0, a uniform heat flux, q_s'', is applied to the outer surface. The inner surface of the shell is insulated. The exact solution to this problem has been programmed by Thorn.[4] Table 11 presents a comparison between the exact solution and the results of SODDIT for the geometry and material properties given in Figure 9.

Table 9. Results for an Infinite Cylinder with Energy Generation and Convection and the Input Data File Used by SODDIT to Compute the Results. ($\Delta r = .05$, $\Delta t = .001$, fully implicit time integration with FCV method.)

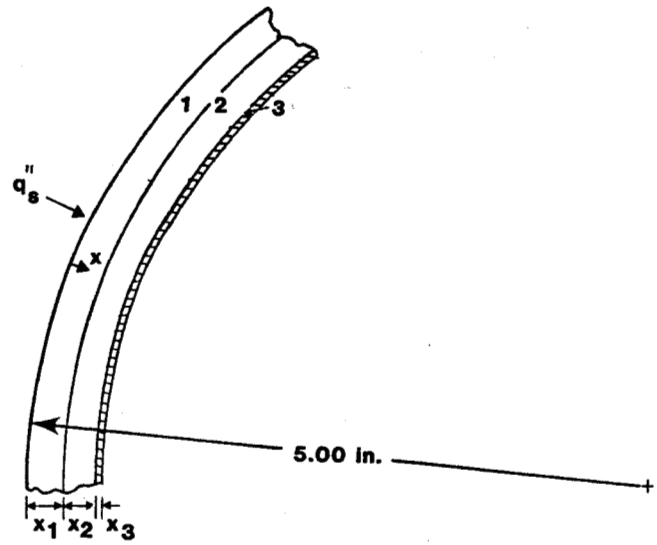
	t = 0.1		t = 1.0	
	Exact	SODDIT	Exact	SODDIT
$q_s''(t)$	.07836725	-.078307	.3983265	-.39818
Depth	T	T	T	T
.0000	.07837	7.8307303E-02	.39833	3.9817693E-01
.0500	.08200	8.1938375E-02	.41769	4.1754304E-01
.1000	.08510	8.5041094E-02	.43597	4.3582075E-01
.1500	.08774	8.7675875E-02	.45318	4.5302414E-01
.2000	.08996	8.9899161E-02	.46932	4.6916684E-01
.2500	.09183	9.1763189E-02	.48441	4.8426195E-01
.3000	.09338	9.3315850E-02	.49847	4.9832203E-01
.3500	.09466	9.4600628E-02	.51150	5.1135906E-01
.4000	.09571	9.5656619E-02	.52352	5.2338439E-01
.4500	.09657	9.6518628E-02	.53454	5.3440871E-01
.5000	.09727	9.7217321E-02	.54457	5.4444205E-01
.5500	.09783	9.7779430E-02	.55361	5.5349372E-01
.6000	.09827	9.8228005E-02	.56168	5.6157234E-01
.6500	.09863	9.8582674E-02	.56878	5.6868585E-01
.7000	.09890	9.8859932E-02	.57493	5.7484152E-01
.7500	.09911	9.9073419E-02	.58011	5.8004610E-01
.8000	.09927	9.9234197E-02	.58435	5.8430602E-01
.8500	.09938	9.9351008E-02	.58764	5.8762806E-01
.9000	.09946	9.9430553E-02	.58999	5.9002120E-01
.9500	.09950	9.9477904E-02	.59139	5.9150432E-01
1.0000	.09952	9.9498566E-02	.59186	5.9216732E-01

```
000000000 310000000 0000000001000000000 000000000 000000000 000000000 0000000000
      SOLID CYLINDER INITIALLY AT UNIFORM TEMPERATURE
      CONVECTION AT THE SURFACE
      UNIFORM ENERGY GENERATION
        0.        1.
        .1
      .001      .001        1.        1.        0.        1.        1.
        1.
 0        0.        1.        1.        0.
 1       10.        1.        1.        0.
        0.
    1   20  1.0
0
        0.        0.    1    3
        0.        0.    2    3
 0      0.        0.        0.        1.
-1     10.        0.        0.        1.
 0      0.        0.        0.        0.
+1     10.        0.        0.        0.
  1.0       0.0       0.0       0.0       0.0          1
```

Table 10. Results for a Solid Sphere with Energy Generation and Convection and the Input Data File Used by SODDIT to Compute the Results. ($\Delta r = .05$, $\Delta t = .001$, fully implicit time integration with FCV method.)

	t = 0.1		t = 1.0	
	Exact	SODDIT	Exact	SODDIT
$q_s''(t)$	.07621169	-.076115	.3054739	-.30536
Depth	T	T	T	T
.0000	.07621	7.6114512E-02	.30547	3.0536237E-01
.0500	.07977	7.9677235E-02	.32035	3.2024099E-01
.1000	.08287	8.2776696E-02	.33443	3.3432428E-01
.1500	.08556	8.5458650E-02	.34771	3.4761706E-01
.2000	.08786	8.7766687E-02	.36021	3.6012400E-01
.2500	.08984	8.9741963E-02	.37193	3.7184968E-01
.3000	.09152	9.1422981E-02	.38287	3.8279851E-01
.3500	.09294	9.2845428E-02	.39304	3.9297474E-01
.4000	.09413	9.4042067E-02	.40244	4.0238247E-01
.4500	.09513	9.5042682E-02	.41107	4.1102559E-01
.5000	.09596	9.5874084E-02	.41893	4.1890783E-01
.5500	.09664	9.6560150E-02	.42604	4.2603274E-01
.6000	.09720	9.7121908E-02	.43239	4.3240373E-01
.6500	.09765	9.7577656E-02	.43799	4.3802410E-01
.7000	.09801	9.7943093E-02	.44284	4.4289712E-01
.7500	.09830	9.8231481E-02	.44694	4.4702624E-01
.8000	.09852	9.8453809E-02	.45029	4.5041546E-01
.8500	.09868	9.8618990E-02	.45289	4.5307024E-01
.9000	.09879	9.8734131E-02	.45475	4.5499998E-01
.9500	.09885	9.8805144E-02	.45587	4.5622671E-01
1.0000	.09887	9.8838625E-02	.45624	4.5681634E-01

```
000000000 310000000 0000000001000000000 000000000 000000000 000000000 0000000000
      SOLID SPHERE INITIALLY AT UNIFORM TEMPERATURE
      CONVECTION AT THE SURFACE
      UNIFORM ENERGY GENERATION
        Ø.        1.
        .1
      .ØØ1      .ØØ1        1.        1.        Ø.        1.        2.
        1.
 Ø       Ø.        1.        1.        Ø.
 1      1Ø.        1.        1.        Ø.
        Ø.
    1   2Ø  1.Ø
Ø
        Ø.        Ø.    1    3
        Ø.        Ø.    2    3
 Ø      Ø.        Ø.        Ø.        1.
-1     1Ø.        Ø.        Ø.        1.
 Ø      Ø.        Ø.        Ø.        Ø.
+1     1Ø.        Ø.        Ø.        Ø.
   1.Ø     Ø.Ø        Ø.Ø       Ø.Ø       Ø.Ø         1
```



a_1 = 4.05	ft^2/hr	k_1 = 55.8	Btu/ft-hr-R
a_2 = 0.02136	ft^2/hr	k_2 = 0.0292	Btu/ft-hr-R
a_3 = 2.03	ft^2/hr	k_3 = 82.26	Btu/ft-hr-R

x_1 = 0.25 inch

x_2 = 0.25 inch $\qquad$ q''_s = 36,000 Btu/ft^2-hr

x_3 = 0.02 inch $\qquad$ T_1 = 536R

Figure 9. Geometry and Material Properties for a Composite Cylindrical Shell. It can be assumed that ρ_1, ρ_2, and ρ_3 are arbitrary constant values (e.g., $\rho_1 = \rho_2 = \rho_3 = 1.0$ lb_m/ft^3).

Table 11. Comparison of Temperature Profile in a Composite Cylindrical Shell Computed by the Exact Solution and by SODDIT. The values shown are in °R and were computed with the data shown in Figure 9. SODDIT used the fully implicit method of time integration ($\theta = 1.$) with lumped capacitance and $\Delta t = 0.1$ s. Materials 1 and 2 were each divided into 20 elements of equal width and material 3 was divided into 10 equal width elements. The depth indicated is below the heated surface.

Time	Depth = 0.0		Depth = 0.25 in.		Depth = 0.5 in.	
(s)	Analytical	SODDIT	Analytical	SODDIT	Analytical	SODDIT
0.0	536.000	536.000	536.000	536.000	536.000	536.000
10.0	888.446	888.592	881.400	881.545	537.200	536.760
20.0	1229.095	1229.21	1221.940	1222.05	546.257	545.982
30.0	1564.880	1564.96	1557.639	1557.72	569.867	569.733
40.0	1896.513	1896.56	1889.194	1889.24	608.862	608.842
50.0	2224.348	2224.37	2216.958	2216.98	662.657	662.735
60.0	2548.650	2548.65	2541.192	2541.19	730.398	730.559
70.0	2869.648	2869.62	2862.127	2862.10	811.215	811.447
80.0	3187.554	3187.51	3179.975	3179.93	904.276	904.570
90.0	3502.556	3502.51	3494.932	3494.87	1008.802	1009.15
100.0	3814.869	3814.80	3807.183	3807.11	1124.061	1124.45

```
0000000001300000000 000000000 000000000 000000000 000000000 000000000 0000000000
        THREE LAYER CYLINDER WITH UNIFORM FLUX AT OUTER SURFACE.
        COMPARISON WITH ANALYTICAL SOLUTION OF TWIG CODE.
        THE LUMPED CAPACITANCE METHOD IS USED.
  Ø.Ø       100.
  10.
  Ø.1       Ø.1       1.Ø       5Ø.                5.        1.
    1.                                      MATERIAL 1
   Ø.Ø      13.7777778  .Ø155
-16000.     13.7777778  .Ø155
    1.                                      MATERIAL 2
   Ø.Ø      1.3670412 8.11111E-6
-16000.     1.3670412 8.11111E-6
    1.                                      MATERIAL 3
   Ø.Ø      4Ø.522167   .Ø2285
+16000.     4Ø.522167   .Ø22850
  536.
    1   2Ø  Ø.25
    2   2Ø  Ø.25
    3   1Ø  Ø.Ø2
Ø
  1.                      1    3
  Ø.Ø                     2    3
   Ø.Ø                  1Ø.
-1 1ØØ.                 1Ø.
   Ø.Ø
+1 1ØØ.
```

Plane Wall with Temperature Dependent Conductivity and Specific Heat

All of the examples of direct thermal diffusion problems considered to this point have had material properties independent of temperature. This subsection examines the ability of SODDIT to accurately solve a transient plane-wall problem whose thermal conductivity and specific heat are linearly related to temperature.

The problem is governed by

$$\rho C_p(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] \quad 0 < x < L, \quad t > 0 \qquad (5\text{-}20)_1$$

with

$$T(x,0) = T_o,\ 0 \le x \le L \qquad (5\text{-}20)_2$$

$$T(0,t) = T_w,\ t > 0 \qquad (5\text{-}20)_3$$

$$T_x(L,t) = 0,\ t > 0, \qquad (5\text{-}20)_4$$

and the thermal property variations given by

$$C_p(T) = C_r\,[1+\beta(T - T_r)] \qquad (5\text{-}21)_1$$

$$k(T) = k_r[1 + \beta(T - T_r)]\ . \qquad (5\text{-}21)_2$$

The width of the wall is L and the material's density is a constant, ρ. The values of T_r, β, C_r, and k_r are determined for the specific material comprising the wall.

The solution of problem (5-20) with (5-21) is found by first introducing Kirchhoff's transformation (Reference 5, p. 129ff),

$$\theta(x,t) = \frac{1}{k_r}\int_{T_r}^{T(x,t)} k(T)dT \qquad (5\text{-}22)$$

giving

$$\theta(x,t) = (T - T_r) + \frac{\beta}{2}(T - T_r)^2\ . \qquad (5\text{-}23)$$

Then solving Eq. (5-23) for T gives

$$T = T_r - \frac{1}{\beta} + \left(\frac{1}{\beta^2} + \frac{2\theta}{\beta}\right)^{1/2}$$

where θ is now the solution of

$$\frac{\partial\theta}{\partial t} = \alpha_r\frac{\partial^2\theta}{\partial x^2} \quad 0 < x < L, \quad t > 0 \qquad (5\text{-}24)_1$$

with

$$\theta(x,0) = \theta_0 \equiv T_o - T_r + \frac{\beta}{2}(T_o - T_r)^{1/2},\ 0 \le x \le L \qquad (5\text{-}24)_2$$

$$\theta(0,t) = \theta_w \equiv T_w - T_r + \frac{\beta}{2}(T_w - T_r)^{1/2}, t > 0 \qquad (5\text{-}24)_3$$

$$\theta_x(L,t) = 0 \qquad (5\text{-}24)_4$$

and

$$\alpha_r \equiv k_r/\rho C_r\ .$$

The solution to Eq. (5-24) can be written (Reference 3, p. 101, or Reference 5, p. 273ff) as

$$\frac{\theta(x,t) - \theta_w}{\theta_0 - \theta_w} = 2\sum_{n=1}^{\infty}\frac{\sin(b_n\,x/L)}{b_n}\exp[-\alpha_r(b_n/L)^2t] \qquad (5\text{-}25)_1$$

where $b_n = (2n - 1)\pi/2$ are the eigenvalues. $(5\text{-}25)_2$

Then, the surface energy flux at x = 0 is

$$q_s''(t) = -\frac{2k_r(\theta_0 - \theta_w)[1+\beta(T_w - T_r)]}{\beta L[1/\beta^2 + 2\theta(0,t)/\beta]^{1/2}} \times \sum_{n=1}^{\infty}\exp[-\alpha_r(b_n/L)^2t]\ . \qquad (5\text{-}26)$$

For purposes of comparison, the above solution and the numerical data from SODDIT solving Eq. (5-20, 21) directly were evaluated with:

$T_o = 536°R$	$\rho = 112.3\ lb_m/ft^3$
$T_w = 3{,}000°R$	$L = 0.1$ in
$k_r = 0.0017$ Btu/s-ft-°R	$T_r = 500°R$
$C_r = 0.425$ Btu/lb_m-°R	$\beta = 4.0765 \times 10^{-4}\ 1/°R$

These results are shown in Table 12.

Table 12. Results for a Plane Wall with Temperature-Dependent Conductivity and Specific Heat Comparing the Exact Solution with the Solution Found Using SODDIT. The discretized domain was divided into 20 equal elements and a time step of 0.001 s was used. Time integration used the fully implicit method ($\theta = 1.$) with the finite control-volume method for element formulation. The input data file is also shown.

	t = 0.1s		t = 1.0s	
	Exact	SODDIT	Exact	SODDIT
$\ddot{q}_s(t)$	1899.47	1949.0 Btu/s-ft^2	430.18	432.87 Btu/s-ft^2
Depth, inches	T,°R	T,°R	T,°R	T,°R
.0000	3000.00	3.00000E+03	3000.00	3.00000E+03
.0050	2764.77	2.76324E+03	2947.56	2.94742E+03
.0100	2523.30	2.52018E+03	2894.88	2.89459E+03
.0150	2280.50	2.27587E+03	2842.28	2.84185E+03
.0200	2041.57	2.03561E+03	2790.10	2.78952E+03
.0250	1811.69	1.80471E+03	2738.67	2.73795E+03
.0300	1595.73	1.58810E+03	2688.35	2.68749E+03
.0350	1397.89	1.39007E+03	2639.49	2.63848E+03
.0400	1221.46	1.21388E+03	2592.44	2.59129E+03
.0450	1068.50	1.06156E+03	2547.55	2.54627E+03
.0500	939.76	9.33747E+02	2505.16	2.50375E+03
.0550	834.67	8.29742E+02	2465.62	2.46409E+03
.0600	751.50	7.47695E+02	2429.24	2.42761E+03
.0650	687.70	6.84940E+02	2396.34	2.39460E+03
.0700	640.26	6.38391E+02	2367.20	2.36537E+03
.0750	606.08	6.04913E+02	2342.07	2.34016E+03
.0800	582.27	5.81617E+02	2321.19	2.31921E+03
.0850	566.34	5.66047E+02	2304.74	2.30271E+03
.0900	556.35	5.56283E+02	2292.87	2.29081E+03
.0950	550.91	5.50961E+02	2285.71	2.28362E+03
.1000	549.19	5.49276E+02	2283.32	2.28122E+03

```
0000000001310000000 000000000 000000000 000000000 000000000 000000000 000000000
  PLANE WALL WITH TEMPERATURE DEPENDENT CONDUCTIVITY AND
  HEAT CAPACITY.  THE WALL IS INITIALLY AT T0.  THE WALL AT X=0 IS SUDDENLY
  RAISED TO TI AND THE BACK WALL IS INSULATED.  THE FCV METHOD IS USED.
        0.       1.1
        .1
      .001      .001        1.
     112.3
 0     500.      .425     .0017
 1    3000.      .858    .00343
      536.
    1   20  0.1
0
        0.        0.    1    2
        0.        0.    2    3
 0      0.     3000.       0.
-1      1.     3000.       0.
 0      0.       0.        0.        0.
+1      1.       0.        0.        0.
```

Solid Sphere, No Generation, Infinite Biot Number

A solid sphere with radius, a, is initially at a uniform temperature T_0. At time t = 0 the surface temperature is suddenly changed to T_s and held at that value for all time [infinite Biot number (= ha/k) boundary]. The temperature history inside the solid sphere is determined by the solution to the following equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\partial}\frac{\partial T}{\partial t} \qquad \textbf{(5-27)}$$

over the domain $0 < r < a$ and $t > 0$.

The initial and boundary conditions are expressed as:

$$T(r,0) = T_0, \qquad 0 \le r \le a \qquad \textbf{(5-28)}_1$$

$$T(a,t) = T_s, \qquad t > 0 \qquad \textbf{(5-28)}_2$$

$$\frac{\partial T(0,t)}{\partial r} = 0, \qquad t \ge 0 \qquad \textbf{(5-28)}_3$$

The solution to Eqs. (5-27, 28) is given (Reference 6, p. 500) as

$$T(r,t) = \sum_{n=1}^{\infty} \frac{2\sin(b_n r)(\sin b_n - b_n\cos b_n)}{b_n r(b_n - \sin b_n\cos b_n)} \exp(-b_n^2 F_0) \qquad \textbf{(5-29)}$$

where $T_0 = 1$ and $T_s = 0$, and the eigenvalues b_n are the solutions to $b_n = (1 - H)\tan b_n$ where H is the Biot number. For $H = \infty$, $b_n = n\pi (n = 1,2,3,\ldots)$. The Fourier number is $F_0 = \alpha t/a^2$, which for the special case $\alpha = a = k = 1$ reduces to time t, and under these conditions, the series of Eq. (5-29) reduces to

$$T(r,t) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{nr}\sin(n\pi r)e^{-n^2\pi^2 t} \; . \qquad \textbf{(5-30)}$$

Differentiating Eq. (5-30) above and setting r = 1 gives the unit area surface flux (remembering k = 1):

$$q_s''(t) = 2\sum_{n=1}^{\infty} e^{-n^2\pi^2 t} \; , \qquad \textbf{(5-31)}$$

which for $T_0 = 1$ and $T_s = 0$ gives a positive flux in the r-direction, which is in agreement with the Second Law of Thermodynamics.

The above equations were programmed to yield the results under the heading EXACT in Table 13, together with the results for the SODDIT code for the same physical conditions and properties. The table shows that a solid spherical geometry without generation is a severe test of the code. The results shown were generated for REQ = $(1/4)^{1/3}$ at the center. Some improvement is gained by going from 20 to 100 elements.

The spherical geometry represents the most severe geometrical test of the SODDIT code because the area is proportional to the square of the radius. Although a wide variety of difference methods will give acceptable results for planar geometry, they may not work as well for cylindrical and spherical geometries. Calculations have been performed for solid spheres of various diameters to investigate the effect of discretization techniques on the accuracy of the temperature predictions. The sphere was assumed to have a uniform initial temperature, with the surface temperature suddenly changed at time zero (infinite Biot number) as described in the opening remarks of this section. Figure 10 compares the dimensionless center temperature determined by SODDIT with the analytical solution given by Eq. (5-30), for a Fourier number of $F_0 = \alpha t/a^2 = 0.12$. The figure shows three distinct curves: curve (a) is the result of using 13 elements while increasing the sphere radius by increasing the element size, curve (b) is the result when the element size is held fixed at 0.002 ft and the radius of the sphere is increased, and curve (c) is the result when the number of elements is fixed at 65 while increasing the radius by increasing the element size. Increasing the number of elements for a given sphere radius improves the accuracy, as one would expect. For a given number of elements, the accuracy increases with increasing radius, but only up to a point; a further increase in radius has no effect. These results suggest that the user of the code should consider performing the same kind of experimental calculations for his particular application to determine a satisfactory discretization scheme.

Table 13. Results for a Solid Sphere, No Energy Sources, Uniform Initial Temperature of 1.0, Infinite Biot Number, Sudden Change of Surface to 0.0 Temperature; Together with SODDIT Input Table for $\Delta t = .001$ with 20 (and 100) Elements

	t = 0.1			t = 0.5		
	Exact	SODDIT		Exact	SODDIT	
		20 elements	100 elements		20 elements	100 elements
$\ddot{q}_s(t)$	.748286	-.82876	-.79891	.0143838	-.015572	-.014887
Depth	T	T	T	T	T	T
0.00	.00000	9.94862E-20	9.94122E-20	.00000E+00	1.86837E-21	1.85244E-21
0.05	.04108	4.14378E-02	4.14368E-02	.75393E-03	7.78589E-04	7.72641E-04
0.10	.08551	8.62272E-02	8.62297E-02	.15720E-02	1.62335E-03	1.61104E-03
0.15	.13261	1.33693E-01	1.33701E-01	.24454E-02	2.52512E-03	2.50608E-03
0.20	.18167	1.83095E-01	1.83106E-01	.33640E-02	3.47359E-03	3.44743E-03
0.25	.23192	2.33654E-01	2.33664E-01	.43167E-02	4.45726E-03	4.42375E-03
0.30	.28258	2.84578E-01	2.84578E-01	.52915E-02	5.46401E-03	5.42284E-03
0.35	.33289	3.35085E-01	3.35065E-01	.62761E-02	6.48091E-03	6.43184E-03
0.40	.38209	3.84430E-01	3.84377E-01	.72573E-02	7.49463E-03	7.43744E-03
0.45	.42950	4.31922E-01	4.31819E-01	.82220E-02	8.49160E-03	8.42611E-03
0.50	.47449	4.76939E-01	4.76770E-01	.91570E-02	9.45825E-03	9.38430E-03
0.55	.51651	5.18939E-01	5.18684E-01	.10049E-01	1.03812E-02	1.02987E-02
0.60	.55508	5.57464E-01	5.57102E-01	.10886E-01	1.12477E-02	1.11564E-02
0.65	.58982	5.92142E-01	5.91648E-01	.11655E-01	1.20455E-02	1.19452E-02
0.70	.62042	6.22682E-01	6.22027E-01	.12347E-01	1.27633E-02	1.26538E-02
0.75	.64662	6.48866E-01	6.48015E-01	.12950E-01	1.33910E-02	1.32720E-02
0.80	.66826	6.70544E-01	6.69448E-01	.13456E-01	1.39200E-02	1.37907E-02
0.85	.68520	6.87629E-01	6.86215E-01	.13857E-01	1.43431E-02	1.42025E-02
0.90	.69735	7.00099E-01	6.98244E-01	.14148E-01	1.46557E-02	1.45012E-02
0.95	.70466	7.08048E-01	7.05499E-01	.14325E-01	1.48566E-02	1.46828E-02
1.00	.70710	7.11874E-01	7.08037E-01	.14384E-01	1.49538E-02	1.47466E-02

```
000000000 310000000 000100000 000000000 000000000 000000000 000000000 0000000000
     A SOLID SPHERE, RO=1.0,K=1.0,RHO=5.0,CP=0.2, WITH THE OUTER SURFACE
     TEMP=0.0, UNIFORM INIT TEMP=1.0, INF BIOT NO.  COMPARE WITH SPHERE
     TRANSIENT ANALYSIS.  DEC. 1985   DATA IN SPHR4T  (NO GAPS PRESENT)
    0.0     0.1000     0.5000
  0.100     0.4000
 0.00       0.050      0.100      0.150      0.200      0.250      0.300
 0.001      0.001        1.0       20.0     1.73E-09  1.00000       2.0
      5.      0.0                                          MAT1
0    0.00       0.200     1.000
+1  1.00        0.200     1.000
    1.00
     1    20  1.0
0
     0.0        0.0        1     2
     0.0        0.0        2     3
0    0.0       0.000       0.0        0.0
-1   0.5       0.000       0.0        0.0
0    0.0        0.00       0.0        0.0
+1   0.5        0.00       0.0        0.0
```

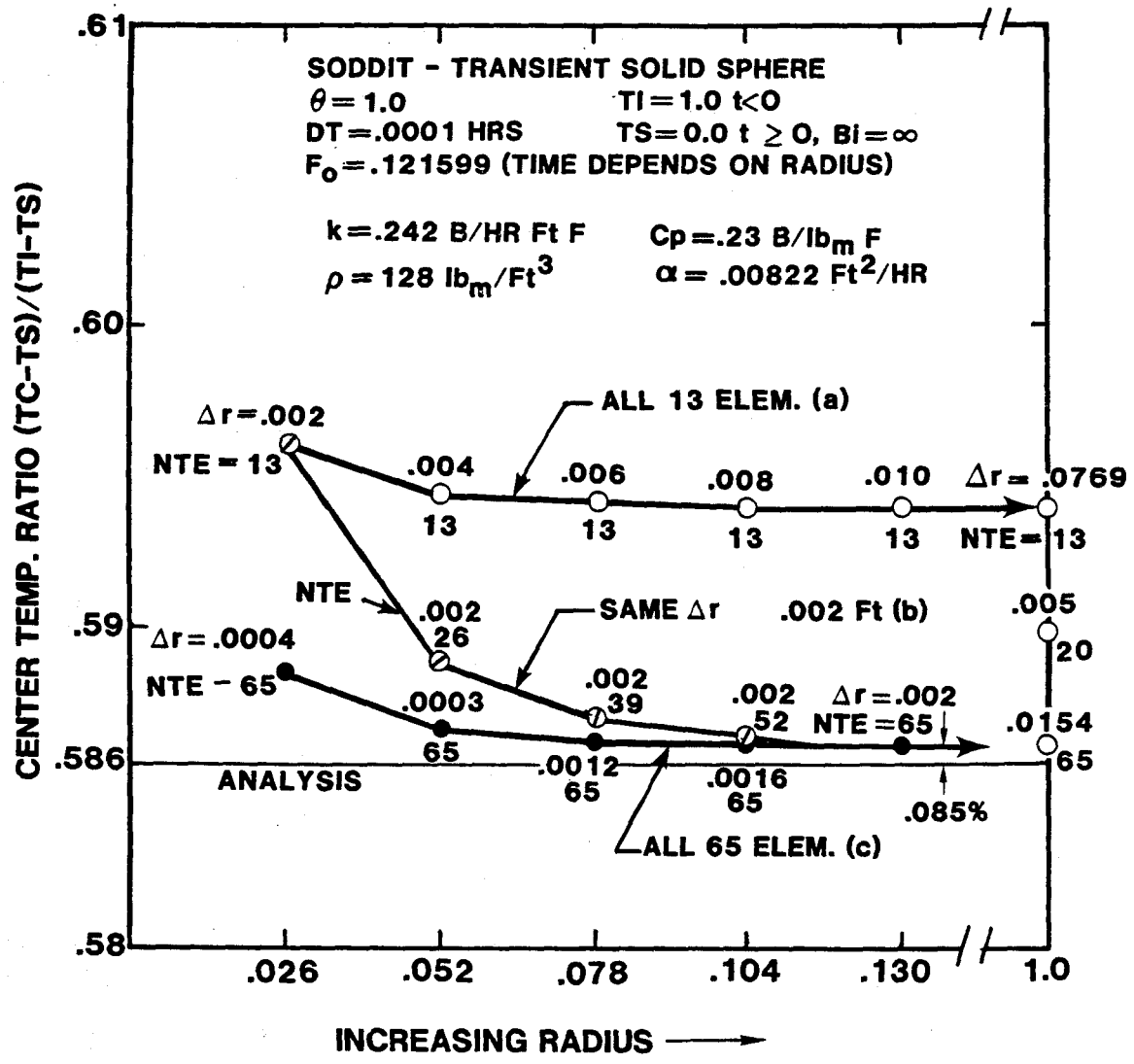


Figure 10. Influence of Radius on Center Temperatures of a Solid Sphere. Bi = ∞, Fo = .121599; (a) 13 elements (increasing Δr), (b) same Δr = 0.02 ft. (increasing elements), (c) all with 65 elements and increasing Δr.

Solid Cylinder with No Generation

A solid cylinder of infinite length (or with the ends insulated) having a radius, a, is initially at a uniform temperature T_0. At time t = 0 the surface temperature is suddenly changed to the environment temperature $T_s = T_\infty$ (infinite Biot number) and held there for all time. The Fourier equation governs the temperature profile and, for the axisymmetric geometry under consideration, is given by:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t};\ 0 \le r \le a,\ t > 0 \qquad \textbf{(5-32)}_1$$

with

$$T(r,0) = T_0,\quad 0 \le r \le a \qquad \textbf{(5-32)}_2$$

$$T(a,t) = T_s,\quad t > 0 \qquad \textbf{(5-32)}_3$$

$$\frac{\partial T(0,t)}{\partial r} = 0,\quad t \ge 0. \qquad \textbf{(5-32)}_4$$

For the case where $a = k = \alpha = 1$ with $T_0 = 1.0$ and $T_s = 0.0$, the exact solution (Reference 6, p. 497) is given as

$$T(r,t) = 2\sum_{n=1}^{\infty} e^{-\lambda_n^2 t} J_0(\lambda_n r)/[\lambda_n J_1(\lambda_n)]\ , \qquad \textbf{(5-33)}$$

and the normalized flux can be determined as

$$q_s''(t)/kA = 2\sum_{n=1}^{\infty} e^{-\lambda_n^2 t} \qquad \textbf{(5-34)}$$

where the eigenvalues λ_n are the solutions to the equation $J_0(\lambda_n) = 0$. The quantities J_0 and J_1 are the zeroth and first-order Bessel functions, respectively. The reader will note that for $\alpha = k = a = 1$, the Fourier number $F_0 = \alpha t/a^2 = t$.

Table 14 compares the results calculated from equations (5-33) and (5-34) under the heading EXACT with those calculated using the SODDIT code using 20 and 100 elements. The input data for the SODDIT code is also shown in the table.

Table 14. Results for an Infinite Solid Cylinder, No Generation. $\Delta r = .05$ and $.001$ with $\Delta t = .001$, fully implicit time integration with the FCV method.

	t = 0.1			t = 0.5		
	Exact	SODDIT		Exact	SODDIT	
		20 elements	100 elements		20 elements	100 elements
$q_s''(t)$:	1.21779	-1.2514	-1.2304	0.11097	-0.11475	-0.11247
B/hr-ft^2						
Depth	T(r,t)	T(r,t)	T(r,t)	T(r,t)	T(r,t)	T(r,t)
0.00	0.	7.6877E-20	7.6931E-20	0.	7.0448E-21	7.0317E-21
0.05	6.2263E-02	6.2571E-02	6.2598E-02	5.6786E-03	5.7376E-03	5.7266E-03
0.10	1.2666E-01	1.2727E-01	1.2732E-01	1.1580E-02	1.1700E-02	1.1678E-02
0.15	1.9223E-01	1.9312E-01	1.9321E-01	1.7647E-02	1.7830E-02	1.7796E-02
0.20	2.5803E-01	2.5917E-01	2.5927E-01	2.3819E-02	2.4066E-02	2.4020E-02
0.25	3.2313E-01	3.2445E-01	3.2457E-01	3.0034E-02	3.0346E-02	3.0288E-02
0.30	3.8663E-01	3.8808E-01	3.8821E-01	3.6230E-02	3.6606E-02	3.6536E-02
0.35	4.4774E-01	4.4925E-01	4.4937E-01	4.2342E-02	4.2783E-02	4.2699E-02
0.40	5.0575E-01	5.0726E-01	5.0736E-01	4.8307E-02	4.8812E-02	4.8715E-02
0.45	5.6008E-01	5.6151E-01	5.6159E-01	5.4063E-02	5.4630E-02	5.4520E-02
0.50	6.1025E-01	6.1155E-01	6.1159E-01	5.9550E-02	6.0178E-02	6.0053E-02
0.55	6.5590E-01	6.5704E-01	6.5703E-01	6.4709E-02	6.5396E-02	6.5256E-02
0.60	6.9680E-01	6.9774E-01	6.9767E-01	6.9486E-02	7.0229E-02	7.0074E-02
0.65	7.3280E-01	7.3353E-01	7.3339E-01	7.3830E-02	7.4627E-02	7.4455E-02
0.70	7.6384E-01	7.6437E-01	7.6415E-01	7.7694E-02	7.8542E-02	7.8352E-02
0.75	7.8993E-01	7.9029E-01	7.8997E-01	8.1037E-02	8.1934E-02	8.1724E-02
0.80	8.1112E-01	8.1136E-01	8.1092E-01	8.3823E-02	8.4766E-02	8.4534E-02
0.85	8.2749E-01	8.2767E-01	8.2709E-01	8.6022E-02	8.7011E-02	8.6752E-02
0.90	8.3911E-01	8.3935E-01	8.3856E-01	8.7609E-02	8.8648E-02	8.8354E-02
0.95	8.4605E-01	8.4656E-01	8.4542E-01	8.8569E-02	8.9671E-02	8.9324E-02
1.00	8.4836E-01	8.4977E-01	8.4776E-01	8.8890E-02	9.0131E-02	8.9658E-02

Table 14. concluded

```
000000000 310000000 000100000 000000000 000000000 000000000 000000000 0000000000
     A SOLID CYLIND, RO=1.0,K=1.0,RHO=5.0,CP=0.2, WITH THE OUTER SURFACE
     TEMP=0.0, UNIFORM INIT TEMP=1.0, INF BIOT NO.  COMPARE WITH CYLIND
     TRANSIENT ANALYSIS.  DEC. 1985   DATA IN CYLD4T  (NO GAPS PRESENT)
    0.0     0.1000     0.5000
  0.100     0.4000
 0.00       0.200      0.400      0.500      0.600      0.800      1.000
 0.001      0.001        1.0       20.0      1.73E-09  1.00000      1.0
       5.       0.0                                      MAT1
0    0.00       0.200     1.000      0.0
-1 1.00         0.200     1.000      0.0
       5.       0.0                                      MAT2
0    0.00       0.200     1.000      0.0
-1 1.00         0.200     1.000      0.0
       5.       0.0                                      MAT3
0    0.00       0.200     1.000
+1 1.00         0.200     1.000
   1.00
     1    20  1.0
0
     0.0        0.0        1    2
     0.0        0.0        2    3
0    0.0       0.000       0.0        0.0
-1   0.5       0.000       0.0        0.0
0    0.0        0.00       0.0        0.0
+1   0.5        0.00       0.0        0.0
```

Two Concentric Hollow Spherical Shells in Steady State, Separated by a Radiation and Convection Gap

The geometry under consideration is shown in Figure 11 together with designations of the pertinent variables. The thermal situation is steady-state heat transfer ($T_1 > T_4$) for the case where the extreme outer temperature T_1 and extreme inner temperature T_4 are fixed. An analytical solution can be obtained from the energy balance for the steady state thermal situation illustrated in Figure 11; thus

$$q_{k1} = q_{Rad} + q_{conv} = q_{k3} \qquad \textbf{(5-34)}$$

where q_{k1} and q_{k3} are the steady state energy flowing through materials MAT1 and MAT3, respectively, and $q_{Rad} + q_{conv}$ is the sum of radiation and convection heat transfer rates through the gap. The reader will note that equation (5-34) and the development to follow hold for the cylindrical and planar geometries as well. The individual flux terms are given by

$$\begin{aligned} q''_{k1} &= K_{12}\,(T_1 - T_2) \\ q''_{k3} &= K_{34}\,(T_3 - T_4) \\ q''_{rad} &= HR(T_2^4 - T_3^4) \\ q''_{conv} &= KC(T_2 - T_3) \end{aligned} \qquad \textbf{(5-35)}$$

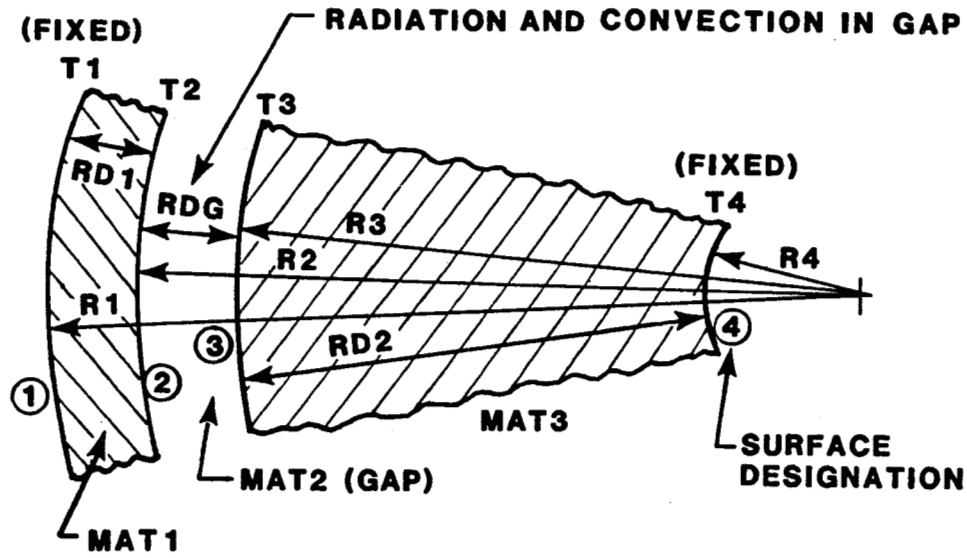


Figure 11. Geometry and Pertinent Variables for Radiation/Convection Gap

The specific form of the conductances K_{12}, K_{34}, HR, and KC depends on the geometry. Equations (5-34) and (5-35) can be combined to yield an expression containing only T_3 such that

$$[AA - (XKK)T_3]^4 - T_3^4 - (C2)T_3 + (CT) = 0 \quad \textbf{(5-36)}$$

where

$$AA = T_1 + (XKK)T_4$$

$$XKK = (K_{34})/(K_{12})$$

$$C2 = CR(XKK) + CR + R6 \quad \textbf{(5-37)}$$

$$CT = CR(T_1) + [CR(XKK) + R6]T_4$$

$$CR = KC/HR \text{ and } R6 = K_{34}/KC \ .$$

The basic units employed in the constants for equation (5-36) must be such that the dimensions of all terms are degrees to the fourth power (an excellent check on algebraic manipulation).

If we for the moment let $Z = T_3$ then equation (5-36) can be written as

$$f(Z) = (a - bZ)^4 - Z^4 - CZ + d = 0 \quad \textbf{(5-38)}$$

where a, b, c, and d are the numerical constants that result when physical properties and dimensions are substituted into Eq. (5-37). The derivative $f'(Z)$ is easily formed so that a Newton-Raphson procedure can be used to obtain T3; thus

$$Z_{j+1} = Z_j - f(Z_j)/f'(Z_j) \quad \textbf{(5-39)}$$

It should be pointed out that the fourth power makes f(Z) very large, even near T_3 and convergence of Eq. (5-39) will be obtained at values of f(Z) near 10^6 or 10^8 rather than zero.

Table 15 lists the dimensions and physical properties employed in the analytical solution and as modeled by SODDIT for fixed temperatures $T_1 = 1132°R$ and $T_4 = 537°R$; Table 16 compares the analytical results with those from SODDIT for 13, 35, 50, and 80 elements in MAT1, 1 element for the gap, and 5, 10, 45, and 60 elements in MAT3; and Table 17 presents a representative input file for those results from SODDIT.

Table 15. Physical Properties, Dimensions, and Temperatures for the Spherical Shells Analysis and SODDIT Model.

Temperatures:	T_1 = 1132R (outside)
	T_4 = 537R (inside)
Dimensions:	RD1 = 0.0026 ft (MAT1)
	RD2 = 0.015625 ft (MAT3)
	RDG = 0.0026 ft (gap)
	R4 = 0.0015 ft
	RADIUS = 0.022325 ft
Properties:	XK1 = 0.2420 B/hr-ft-F (MAT1)
	XK2 = 11.230 B/hr-ft-F (MAT3)
	EMIT1 = 0.73 (dimensionless)
	EMIT2 = 0.45 (dimensionless)
Gap Coefficient:	HCOEF = 8.4245 B/hr-ft^2-F (MAT2)*
	HCOEF = 6.350 B/hr-ft^2-F (ANALYSIS)

*NOTE: SODDIT bases convective conductance for the gap on minimum area, whereas analysis bases convective conductance on maximum area; thus $h_3A_3 = h_2A_2$.

Table 16. Comparison of SODDIT Results for Different Numbers of Elements in MAT1 and MAT 3 with Analysis.

	QCOND	T1	T2	T3	T4
Analysis	2913.9	(1132)†	1096.5671	615.6639	(537)
SODDIT 13,-1,5*	3083.4	(1132)	1094.8424	586.6108	(537)
SODDIT 80,-1,10	2982.7	(1132)	1095.7835	602.5131	(537)
SODDIT 50,-1,45	2925.5	(1132)	1096.5084	614.6794	(537)
SODDIT 35,-1,60	2926.4	(1132)	1096.5338	615.1020	(537)

†Temperatures in parentheses are fixed boundary conditions.

*(number of elements in MAT1), (Gap), (number of elements in MAT3).

Table 17. Representative Input File for Results Shown in Table 16.

```
ØØØØØØØØØ 31ØØØØØØØ ØØØØØØØØØ ØØØØØØØØØ ØØØØØØØØØ ØØØØØØØØØ ØØØØØØØØØ ØØØØØØØØØØ
     TWO CONCENTRIC SPHERES SEP BY A GAP HAVING RAD AND CONVECT, NO GEN,
     INNER AND OUTER SURF TEMPS FIXED. COMPARISON WITH ANALYSIS ZSRCG
     JANUARY 1986,     DATA IS IN SPHR1
    Ø.Ø     Ø.4ØØØ
 Ø.4ØØ
 Ø.Ø2       Ø.Ø2          1.Ø       2Ø.Ø      1.73E-Ø9  Ø.Ø22325     2.Ø
    128.        Ø.Ø         Ø.Ø                            MAT1
Ø   5ØØ.        Ø.23      Ø.242      Ø.73Ø
-1 15ØØ.        Ø.23      Ø.242      Ø.73Ø
    5ØØ.        Ø.Ø         Ø.Ø                            MAT2(GAP)
Ø   5ØØ.       8.424548    Ø.ØØ26     Ø.Ø
-1 15ØØ.       8.424548    Ø.ØØ26     Ø.Ø
    5ØØ.        Ø.Ø         Ø.Ø                            MAT3
Ø   5ØØ.        Ø.11      11.23      Ø.45Ø
+1 15ØØ.        Ø.11      11.23      Ø.45Ø
   5ØØ.
    1   8Ø  Ø.ØØ26
    2   -1  Ø.ØØ26
    3   1Ø  Ø.Ø15625
    Ø
    Ø.Ø         Ø.Ø         1     2
    Ø.Ø         Ø.Ø         2     2
Ø   Ø.Ø        1132.        Ø.Ø        Ø.Ø
-1  Ø.4        1132.        Ø.Ø        Ø.Ø
Ø   Ø.Ø         537.        Ø.Ø        Ø.Ø
+1  Ø.4         537.        Ø.Ø        Ø.Ø
```

Two Concentric Hollow Spherical Shells in Steady State, Separated by a Zero-Thickness Contact Conductance Gap

The geometry under consideration is shown in Figure 12. The designations for the pertinent variables are similar to those shown for the concentric hollow sperical shells separated by a radiation and convection gap, with the exception that in this case RDG = 0.0 (see the previous sample problem).

The thermal situation is steady-state heat transfer ($T_1 > T_4$) with the outer and inner temperatures, T_1 and T_4, fixed.

An analytical solution can be obtained for T_2 and T_3 from the steady-state energy balance

$$q_{k1} = q_{kcond} = q_{k3} \tag{5-40}$$

where

$$\begin{aligned} q_{k1} &= K_{12}(T_1 - T_2) \\ q_{kcond} &= K_c(T_2 - T_3) \\ q_{k3} &= K_{34}(T_3 - T_4). \end{aligned} \tag{5-41}$$

The conductances in Eq. (5-41) depend on the conductivities and dimensions, for example

$$K_{12} = 4\pi(k_1)R1(R2)/(R1 - R2) \tag{5-42}$$

and

$$K_c = HCONT(A2) \tag{5-43}$$

where k_1 is thermal conductivity for MAT1, HCONT is the contact conductance between MAT1 and MAT3, and A2 is the surface area that HCONT acts on. If we let $C34 = K_{34}/KC$ and $C12 = KC/K_{12}$, the conductance ratios, then equating q_{kcond} with q_{k3} yields

$$T_2 = T_3(1 + C34) - C34(T_4) \tag{5-44}$$

and equating q_{k1} with q_{k3} yields

$$T_2 = [T_1 + C12(T_3)]/(1 + C12). \tag{5-45}$$

Equations (5-44) and (5-45) can be solved to give the interface temperature T_3; thus

$$T_3 = \frac{T_1 + (1+C12)C34(T_4)}{(1 + C12)(1 + C34) - C12}. \tag{5-46}$$

Once the temperatures T_2 and T_3 are known, any one of the three expressions in Eq. (5-41) can be employed to determine the heat flux from T_1 to T_4.

Table 18 lists the dimensions and the physical properties used in the analytical solution described above. The value for HCONT was arbitrarily selected to give roughly the same temperature difference across the contact conductance gap as was the case for the previous sample problem. Table 19 summarizes the analytical calculations and compares the results to those from SODDIT. Table 20 gives the actual input data file with the SODDIT code using the format arrangement given in Section 2.

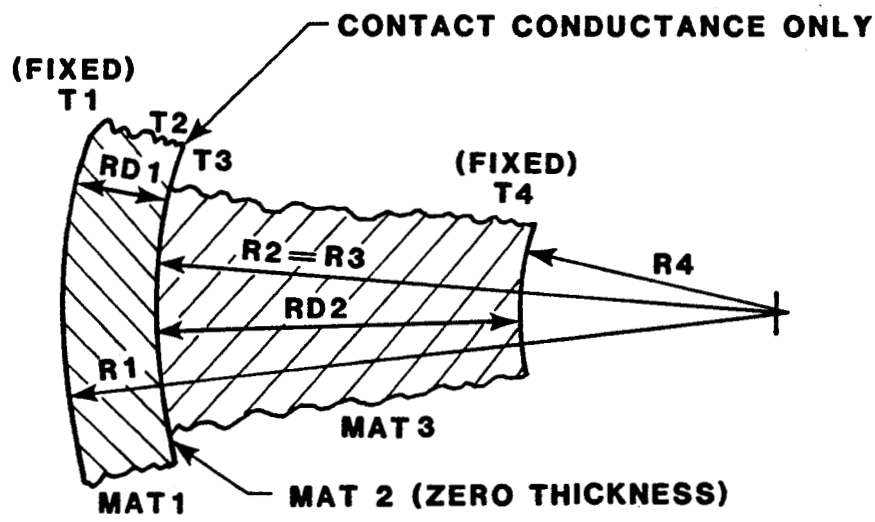


Figure 12. Geometry and Pertinent Variables for Contact Conductance

Table 18. Physical Properties, Temperatures, and Dimensions for the Analysis and SODDIT Model of Figure 12.

Temperatures:	T_1 = 1132R (outside)
	T_4 = 537R (inside)
Dimensions:	RD1 = 0.0026 ft (MAT1)
	RD2 = 0.015625 ft (MAT3)
	RDG = 0.0 ft (MAT2,GAP)
	R4 = 0.015 ft
	R1 = 0.033225 ft (RADIUS)
Properties:	k_1 = 0.242 B/hr-ft-F (MAT1)
	k_3 = 11.23 B/hr-ft-F (MAT3)
	K_{12} = 1.190126071 B/hr-F
	K_{34} = 4.148938055 B/hr-F
	C12 = 0.090558322
	C34 = 38.49600082
Contact Cond.:	KC = 0.10777582 B/hr-F
	HCONT = 9.14448553 B/hr-ft^2-F
	A2 = 0.0117858812 ft^2

Table 19. Comparison of SODDIT Results with Exact Analysis, Two Concentric Spherical Shells with Interface Contact Conductance. Input data in Table 20 for the values of Table 18.

	Analysis	SODDIT†
T_1	(1132)*	(1132)
T_2	1083.741595R	1083.7411R
T_3	550.842961R	550.8359R
T_4	(537)	(537)
QCOND	4140.41 B/hr-ft^2	4145.7

†Using 60 elements in MAT1 and 30 elements in MAT3.

*Numbers in parentheses are fixed boundary temperatures.

Table 20. Input Data File Used to Produce Results from SODDIT Code Shown in Table 19.

```
000000000 310000000 000000000 000000000 000000000 000000000 000000000 0000000000
     TWO CONCENTRIC SPHERES SEP BY A ZERO WIDTH GAP,CONTACT COND ONLY
     INNER AND OUTER SURF TEMPS FIXED. COMPARE ANALYSIS IN NOTEBOOK.
     JANUARY 1985,     DATA IS IN SPHRK
    0.0     0.4000
 0.400
 0.02       0.02          1.0       20.0      1.73E-09  0.033225     2.0
    128.       0.0         0.0                                MAT1
0   500.       0.23       0.242      0.000
-1 1500.       0.23       0.242      0.000
    500.       0.0         0.0                                MAT2(GAP)
0   500.      0.0000     0.0026      9.1444855
-1 1500.      0.0000     0.0026      9.1444855
    500.       0.0         0.0                                MAT3
0   500.       0.11       11.23      0.000
+1 1500.       0.11       11.23      0.000
   500.
    1    60 0.0026
    2    -1 0.0
    3    30 0.015625
0
    0.0        0.0        1    2
    0.0        0.0        2    2
0   0.0       1132.       0.0        0.0
-1  0.2       1132.       0.0        0.0
0   0.0        537.       0.0        0.0
+1  0.2        537.       0.0        0.0
```

Type 1 Front-Face Surface Energy Balance for a Plane Wall

This example makes use of the front-face surface energy balance Type 1 option in which the convective input is given by a recovery enthalpy difference times a transfer coefficient. A planar structure consisting of 0.20 in. of 4340 steel over 0.35 in. of 6061 aluminum is exposed to an aerodynamic heating environment that is time dependent. The steel thermal properties are highly temperature dependent, while those of the aluminum were assumed constant; the detailed property values are given in the input file shown in Table 21. The units used in this example are Btu-R-lbm-ft-sec with the exception that the input nodal information is in inches and the air enthalpy data is in K and cal/gm. The enthalpy data is for air and covers a much wider range of temperature and pressure than is necessary for this problem. Note that at the lower temperatures, the enthalpy is independent of pressure; however, this information must still be included in the table because of the linear interpolation requirements. The back face of the aluminum is perfectly insulated. This example is designed to be representative of the kind of real-world problems that can be solved with SODDIT instead of verifying the accuracy of the code.

Table 21. Input Data File for Type 1 Front-Face Surface Energy Balance for a Plane Wall.

```
0000000001300000000 000000000 000000000 000000000 000000000 000000000 0000000000
    EXAMPLE PROBLEM WITH AERODYNAMIC HEATING TYPE I SURFACE ENERGY BALANCE
    AIR ENTHALPY AS A FUNCTION OF TEMPERATURE AND PRESSURE
    0.20 IN 4340 STEEL, 0.35 IN 6061 ALUMINUM
 4.0        20.        25.0
 2.0         1.0
  0.005     2.0          1.0      10.
  483.6     0.0        536.      4340 STEEL,LLL DATA 11-84
   459.       .120       .00692     .3                4340STEEL,LLL DATA 11-84
   859.       .124       .00679     .3                4340STEEL,LLL DATA 11-84
   1260.      .148       .00620     .3                4340STEEL,LLL DATA 11-84
   1657.      .180       .00517     .3                4340STEEL,LLL DATA 11-84
   1865.      .200       .00431     .3                4340STEEL,LLL DATA 11-84
   1874.     1.648       .00431     .3                4340STEEL,LLL DATA 11-84
   1883.      .15        .00431     .3                4340STEEL,LLL DATA 11-84
   1909.      .15        .00417     .3                4340STEEL,LLL DATA 11-84
   2059.      .14        .00433     .3                4340STEEL,LLL DATA 11-84
   2461.      .145       .00433     .3                4340STEEL,LLL DATA 11-84
-1 5000.      .145       .00433     .3                4340STEEL,LLL DATA 11-84
   169.3                           ALUMINUM 6061 1-4-85
   460.       .23        .027
+1 2000.      .23        .027
  580.
    1    20 0.20        0.002
    2       0.35        0.015      0.025
0
   0.0        0.0          1     1
   0.0        0.0          2     3
   0.0       1.0                    .00001     1.0        10.0
   10.       77.8506                .22545     1.28773    1971.4
   12.      114.5783                .18952     .79801     2595.4
   14.      155.0341                .19196     .70502     3042.2
   16.      197.6355                .17910     .62029     3389.6
   18.      234.2150                .15981     .51462     3699.8
   19.63    240.2509                .13739     .42266     3774.2
   20.      235.1528                .13142     .40140     3745.3
   25.      191.0417                .07055     .20214     3438.6
-1 30.0     163.8670               0.0428      .10923     3251.8
     0.
+1 30.0
 .001              1.0E-9   3000.              1621.35          AIR-NON-ABLATE
 .001              1.0E-9   2800.              1449.78          AIR-NON-ABLATE
 .001              1.0E-9   2600.              1153.40          AIR-NON-ABLATE
 .001              1.0E-9   2400.               834.18          AIR-NON-ABLATE
 .001              1.0E-9   2200.               619.95          AIR-NON-ABLAT
 .001              1.0E-9   2000.               498.29          AIR-NON-ABLATE
 .001              1.0E-9   1800.               419.96          AIR-NON-ABLATE
 .001              1.0E-9   1600.               356.29          AIR-NON-ABLATE
 .001              1.0E-9   1400.               296.92          AIR-NON-ABLATE
 .001              1.0E-9   1200.               239.28          AIR-NON-ABLAT
 .001              1.0E-9   1000.               182.95          AIR-NON-ABLATE
 .001              1.0E-9    800.               128.10          AIR-NON-ABLATE
 .001              1.0E-9    600.                75.487         AIR-NON-ABLATE
 .001              1.0E-9    400.                24.606         AIR-NON-ABLATE
 .001              1.0E-9    300.                 0.533         AIR-NON-ABLATE
 .001              1.0E-9    200.               -23.406         AIR-NON-ABLATE 1
 .01               1.0E-9   2800.              1125.896         AIR-NON-ABLATE
 .01               1.0E-9   2600.               868.233         AIR-NON-ABLATE
 .01               1.0E-9   2400.               685.626         AIR-NON-ABLATE
 .01               1.0E-9   2200.               567.318         AIR-NON-ABLATE
```

Table 21. (continued)

.01	1.0E-9	2000.	484.684	AIR-NON-ABLATE
.01	1.0E-9	1800.	417.450	AIR-NON-ABLATE
.01	1.0E-9	1600.	355.988	AIR-NON-ABLATE
.01	1.0E-9	1400.	296.902	AIR-NON-ABLATE
.01	1.0E-9	1200.	239.282	AIR-NON-ABLATE
.01	1.0E-9	1000.	182.95	AIR-NON-ABLATE
.01	1.0E-9	800.	128.097	AIR-NON-ABLATE
.01	1.0E-9	600.	75.487	AIR-NON-ABLATE
.01	1.0E-9	400.	24.606	AIR-NON-ABLATE
.01	1.0E-9	300.	0.533	AIR-NON-ABLATE
.01	1.0E-9	200.	-23.406	AIR-NON-ABLATE 1
.1	1.0E-9	2800.	887.674	AIR-NON-ABLATE
.1	1.0E-9	2600.	740.271	AIR-NON-ABLATE
.1	1.0E-9	2400.	632.780	AIR-NON-ABLATE
.1	1.0E-9	2200.	550.096	AIR-NON-ABLATE
.1	1.0E-9	2000.	480.345	AIR-NON-ABLATE
.1	1.0E-9	1800.	416.656	AIR-NON-ABLATE
.1	1.0E-9	1600.	355.893	AIR-NON-ABLATE
.1	1.0E-9	1400.	296.897	AIR-NON-ABLATE
.1	1.0E-9	1200.	239.282	AIR-NON-ABLATE
.1	1.0E-9	1000.	182.954	AIR-NON-ABLATE
.1	1.0E-9	800.	128.097	AIR-NON-ABLATE
.1	1.0E-9	600.	75.487	AIR-NON-ABLATE
.1	1.0E-9	400.	24.606	AIR-NON-ABLATE
.1	1.0E-9	300.	0.533	AIR-NON-ABLATE
.1	1.0E-9	200.	-23.406	AIR-NON-ABLATE 1
1.0	1.0E-9	2800.	790.952	AIR-NON-ABLATE
1.0	1.0E-9	2600.	695.606	AIR-NON-ABLATE
1.0	1.0E-9	2400.	615.480	AIR-NON-ABLATE
1.0	1.0E-9	2200.	544.595	AIR-NON-ABLATE
1.0	1.0E-9	2000.	478.972	AIR-NON-ABLATE
1.0	1.0E-9	1800.	416.407	AIR-NON-ABLATE
1.0	1.0E-9	1600.	355.865	AIR-NON-ABLATE
1.0	1.0E-9	1400.	296.896	AIR-NON-ABLATE
1.0	1.0E-9	1200.	239.283	AIR-NON-ABLATE
1.0	1.0E-9	1000.	182.955	AIR-NON-ABLATE
1.0	1.0E-9	800.	128.097	AIR-NON-ABLATE
1.0	1.0E-9	600.	75.487	AIR-NON-ABLATE
1.0	1.0E-9	400.	24.606	AIR-NON-ABLATE
1.0	1.0E-9	300.	0.533	AIR-NON-ABLATE
1.0	1.0E-9	200.	-23.406	AIR-NON-ABLATE 1
10.0	1.0E-9	2800.	758.105	AIR-NON-ABLATE
10.0	1.0E-9	2600.	681.068	AIR-NON-ABLATE
10.0	1.0E-9	2400.	609.959	AIR-NON-ABLATE
10.0	1.0E-9	2200.	542.858	AIR-NON-ABLATE
10.0	1.0E-9	2000.	478.545	AIR-NON-ABLATE
10.0	1.0E-9	1800.	416.334	AIR-NON-ABLATE
10.0	1.0E-9	1600.	355.860	AIR-NON-ABLATE
10.0	1.0E-9	1400.	296.898	AIR-NON-ABLATE
10.0	1.0E-9	1000.	182.956	AIR-NON-ABLATE
10.0	1.0E-9	800.	128.098	AIR-NON-ABLATE
10.0	1.0E-9	400.	24.606	AIR-NON-ABLATE
10.0	1.0E-9	300.	0.533	AIR-NON-ABLATE
10.0	1.0E-9	200.	-23.406	AIR-NON-ABLATE 1
100.	1.0E-9	2800.	747.525	AIR-NON-ABLATE
100.	1.0E-9	2600.	676.462	AIR-NON-ABLATE
100.	1.0E-9	2400.	608.237	AIR-NON-ABLATE
100.	1.0E-9	2200.	542.335	AIR-NON-ABLATE
100.	1.0E-9	2000.	478.432	AIR-NON-ABLATE
100.	1.0E-9	1800.	416.329	AIR-NON-ABLATE

Table 21. (concluded)

100.	1.0E-9	1600.	355.872	AIR-NON-ABLATE
100.	1.0E-9	1400.	296.909	AIR-NON-ABLATE
100.	1.0E-9	1200.	239.291	AIR-NON-ABLATE
100.	1.0E-9	1000.	182.959	AIR-NON-ABLATE
100.	1.0E-9	800.	128.099	AIR-NON-ABLATE
100.	1.0E-9	600.	75.487	AIR-NON-ABLATE
100.	1.0E-9	400.	24.606	AIR-NON-ABLATE
100.	1.0E-9	300.	0.533	AIR-NON-ABLATE
100.	1.0E-9	200.	-23.406	AIR-NON-ABLATE 3

Material interface temperatures as a function of time are presented below. Due to the nonlinearity of this problem, there is no exact solution with which to compare the results.

TIME s	Material Interface Temperatures °R		
4.000E+00	580.0000	580.0000	580.0000
6.000E+00	589.7959	582.8917	582.0744
8.000E+01	603.3991	589.7434	587.9033
1.000E+01	621.5354	599.8175	596.7904
1.200E+01	643.1015	613.7176	609.4667
1.400E+01	672.7121	632.2159	626.4192
1.600E+01	705.6829	655.7437	648.4007
1.800E+01	736.9239	682.5114	674.2688
2.000E+01	755.4951	708.1129	700.4287
2.100E+01	760.3780	718.8905	712.0056
2.200E+01	763.7368	728.0862	722.0861
2.300E+01	765.8864	735.7335	730.5992
2.400E+01	767.1403	742.0890	737.7722
2.500E+01	767.6291	747.2607	743.7010

A Thin, 1-D Fin with Convection From its Surface

A thin fin of constant perimeter P and circular cross-sectional area A_c loses heat from its surface by convection heat transfer to the environment maintained at T_∞. Initially, the fin is at a uniform temperature T_i. Then, one end of the fin has its temperature suddenly dropped to T_0 while the other end is insulated. During the transient phase, energy is conducted along the axis of the fin and is then lost to the environment by convection. If the radial Biot number (=hD/4k) is less than 0.1, there is negligible radial conduction and the conduction/convection process is one-dimensional, in the axial direction. The radial convection energy loss is then modeled as a temperature-dependent energy sink (or source), although the material itself is *not* generating energy.

The mathematical model is then

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} - \frac{4h}{D}(T - T_\infty);\ 0 < x < L,\ t > 0 \qquad (5\text{-}47)_1$$

where

$$T(x,0) = T_i,\ 0 \le x \le L \qquad (5\text{-}47)_2$$

and

$$T(0,t) = T_0,\ t > 0 \qquad (5\text{-}47)_3$$

$$\frac{\partial T(L,t)}{\partial t} = 0,\quad t > 0\ . \qquad (5\text{-}47)_4$$

The exact solution to this problem is

$$T(x,t) = T_\infty + (T_0 - T_\infty)\frac{\cosh\left[w\left(1-\frac{x}{L}\right)\right]}{\cosh w} + 2(T_i - T_\infty)\sum_{n=0}^{\infty}\frac{(-1)^n}{\lambda_n}\left[1-\frac{(T_0-T_\infty)}{(T_i-T_\infty)}\frac{\lambda_n^2}{(w^2+\lambda_n^2)}\right]\cos\left[\lambda_n\left(1-\frac{x}{L}\right)\right]\exp\left[-(w^2+\lambda_n^2)\frac{\alpha t}{L^2}\right] \qquad (5\text{-}48)_1$$

where the eigenvalues are

$$\lambda_n = (2n+1)\pi/2,\quad n=0,1,2,\ldots \qquad (5\text{-}48)_2$$

and

$$w^2 = \frac{4hL^2}{kD},\ \alpha = \frac{k}{\rho C_p}\ . \qquad (5\text{-}48)_3$$

The rate of energy crossing the fixed temperature boundary (i.e., x = 0) is

$$q_0''(t) = -\frac{k(T_i - T_\infty)}{L}\left\{\frac{(T_0-T_\infty)}{(T_i-T_\infty)}\, w \tanh w - 2\sum_{n=0}^{\infty} \exp\left[-(w^2+\lambda_n^2)\frac{\alpha t}{L^2}\right]\left[1 - \frac{(T_0-T_\infty)}{(T_i-T_\infty)}\frac{\lambda_n^2}{(w^2+\lambda_n^2)}\right]\right\} . \quad \textbf{(5-49)}$$

This problem was included as a test of the temperature-dependent energy source additions to SODDIT. The exact solutions above were evaluated and the results shown in Table 22 along with the corresponding predictions from SODDIT. The physical quantities used in evaluating each solution include:

D = 0.05 ft

L = 1.0 ft

k = 42.0 BTU/hr-ft-°F

h = 10.0 BTU/hr-ft^2-°F

ρ = 493.0 lbm/ft^3

C_p = 0.108 BTU/lbm-°F

For these values, the radial Biot number is roughly 3×10^{-3} so that the problem is truely one-dimensional. The exact solution was evaluated with 31 terms in the series and SODDIT had 50 equal elements and a time stepsize of $\Delta t = 0.0001$ hr with fully implicit ($\theta = 1.0$) time marching. The temperature values agree quite well, but the heat flux computations are not as close, although they are within 6% of each other. Using a geometric progression in element size with smaller elements at the x=0 boundary may improve the heat flux agreement.

Inverse Heat Conduction Problems

Plane Wall with Energy Generation

Consider the plane wall of thickness a discussed in the first sample problem. It is assumed that the front face (x = 0) has a surface energy flux and/or temperature to be estimated from the history of a single thermocouple measurement located at x = 0.5 (at the midplane), while the back face (x = 1.0) is insulated. The initial temperature of the wall is T = 0. and at t = 0., a uniform energy generation rate is started. The thermocouple data are simulated by the exact solution history for x = 0.5. The results of the computations of this inverse heat conduction problem from SODDIT are displayed in Table 23 and are therein compared to its exact solution (cf, Eqs. 5-2, -3).

Table 22. Heat Transfer in a Thin, 1-D Fin With Convection From its Surface. The convective loss from the fin is modeled as though it was an energy source dependent on temperature. Fifty elements of equal length were used with $\Delta t = 0.01$. QA = QC = QD = QE = 0, but QB = −1. The FCV method was used. The input data file is also shown. The data shown are for t = 5.0, essentially a steady-state time.

	t = 0.01		t = 0.1	
	Exact	SODDIT	Exact	SODDIT
$\dot{q}''_s$:	5360.759	−5515.8	−2798.461	2643.2
Depth	T	T	T	T
0.0000	70.00000	7.0000000E+01	70.00000	7.0000000E+01
0.0200	72.61148	7.2626573E+01	68.74127	6.8741320E+01
0.0400	75.27229	7.5301145E+01	67.62133	6.7621600E+01
0.0600	77.88675	7.7926753E+01	66.62759	6.6628203E+01
0.0800	80.37222	8.0419865E+01	65.74841	6.5749485E+01
0.1000	82.66320	8.2714625E+01	64.97309	6.4974716E+01
0.1200	84.71369	8.4765174E+01	64.29177	6.4294016E+01
0.1400	86.49755	8.6545949E+01	63.69538	6.3698289E+01
0.1600	88.00710	8.8050131E+01	63.17555	6.3179166E+01
0.1800	89.25034	8.9286673E+01	62.72461	6.2728955E+01
0.2000	90.24722	9.0276468E+01	62.33550	6.2340588E+01
0.2200	91.02572	9.1048225E+01	62.00175	6.2007581E+01
0.2400	91.61793	9.1634580E+01	61.71743	6.1723988E+01
0.2600	92.05687	9.2068801E+01	61.47710	6.1484371E+01
0.2800	92.37388	9.2382309E+01	61.27580	6.1283756E+01
0.3000	92.59700	9.2603045E+01	61.10900	6.1117608E+01
0.3200	92.75005	9.2754643E+01	60.97256	6.0981795E+01
0.3400	92.85238	9.2856219E+01	60.86274	6.0872562E+01
0.3600	92.91907	9.2922635E+01	60.77614	6.0786506E+01
0.3800	92.96143	9.2965022E+01	60.70968	6.0720548E+01
0.4000	92.98767	9.2991430E+01	60.66058	6.0671913E+01
0.4200	93.00351	9.3007495E+01	60.62636	6.0638102E+01
0.4400	93.01283	9.3017041E+01	60.60476	6.0616878E+01
0.4600	93.01818	9.3022581E+01	60.59380	6.0606243E+01
0.4800	93.02117	9.3025723E+01	60.59168	6.0604418E+01
0.5000	93.02280	9.3027464E+01	60.59684	6.0609826E+01
0.5200	93.02367	9.3028408E+01	60.60788	6.0621079E+01
0.5400	93.02412	9.3028907E+01	60.62357	6.0636955E+01
0.5600	93.02434	9.3029166E+01	60.64286	6.0656390E+01
0.5800	93.02446	9.3029297E+01	60.66481	6.0678457E+01
0.6000	93.02451	9.3029362E+01	60.68863	6.0702360E+01
0.6200	93.02453	9.3029394E+01	60.71362	6.0727415E+01
0.6400	93.02455	9.3029409E+01	60.73920	6.0753042E+01
0.6600	93.02455	9.3029416E+01	60.76488	6.0778751E+01
0.6800	93.02455	9.3029419E+01	60.79026	6.0804134E+01
0.7000	93.02455	9.3029420E+01	60.81498	6.0828854E+01
0.7200	93.02456	9.3029421E+01	60.83878	6.0852638E+01
0.7400	93.02456	9.3029421E+01	60.86143	6.0875264E+01
0.7600	93.02456	9.3029421E+01	60.88276	6.0896560E+01

Table 22. (concluded)

	t = 0.01		t = 0.1	
	Exact	SODDIT	Exact	SODDIT
$\dot{q}''_s$:	5360.759	−5515.8	−2798.461	2643.2
Depth	T	T	T	T
0.7800	93.02456	9.3029421E+01	60.90262	6.0916390E+01
0.8000	93.02456	9.3029421E+01	60.92093	6.0934653E+01
0.8200	93.02456	9.3029421E+01	60.93759	6.0951275E+01
0.8400	93.02456	9.3029421E+01	60.95256	6.0966204E+01
0.8600	93.02456	9.3029421E+01	60.96580	6.0979406E+01
0.8800	93.02456	9.3029421E+01	60.97729	6.0990859E+01
0.9000	93.02456	9.3029421E+01	60.98701	6.1000552E+01
0.9200	93.02456	9.3029421E+01	60.99497	6.1008480E+01
0.9400	93.02456	9.3029421E+01	61.00115	6.1014643E+01
0.9600	93.02456	9.3029421E+01	61.00557	6.1019042E+01
0.9800	93.02456	9.3029421E+01	61.00821	6.1021680E+01
1.0000	93.02456	9.3029421E+01	61.00910	6.1022559E+01

```
000000000 310000000 0000000003000000000 000000000 000000000 000000000 000000000
    FIN OF UNIFORM CROSS SECTION WITH T(x,Ø)=To AND WITH T(Ø,t)=TL, q(L,t)=Ø.
    CONVECTION OCCURS ALONG ITS LENGTH WITH TINF=Ti AND IS MODELED AS A
    TEMPERATURE DEPENDENT SOURCE WITH QA=H*P*Ti/AC, QB=-h*P/AC,QC=QD=QE=Ø.
  Ø.        .Ø1       .1
   .Ø1      .99
    .ØØØ1   .ØØØ1      1.Ø
   493.
 Ø   4Ø.     .1Ø8      42.
+1  1ØØ.     .1Ø8      42.
   1ØØ.
    1   5Ø   1.Ø
    Ø
   Ø.         Ø.         1    2
   Ø.         Ø.         2    3
 Ø  Ø.       7Ø.
-1 1Ø.       7Ø.
 Ø  Ø.       Ø.         Ø.        Ø.
+1 1Ø.       Ø.         Ø.        Ø.
4ØØØØ.      -8ØØ.      Ø.        Ø.        Ø.         1
```

Table 23. Inverse Solution for Uniform Energy Generation Within a Plane Wall. The input data file used by SODDIT is also shown. The computed results are compared to the exact solution. The SODDIT solution used $\Delta x = .05$, $\Delta t = .01$, fully implicit time integration with the Finite Control Volume method, 2 future times (NFT = 2), and 2 time integrations per problem time step.

	t = 0.05		t = 0.25	
	Exact	SODDIT	Exact	SODDIT
$q_s''(t)$	.2523133	-.24152	.5622335	-.55598
Depth	T	T	T	T
.00	.00000	1.68559E-03	.00000	1.26818E-03
.05	.01142	1.27007E-02	.02688	2.79066E-02
.10	.02065	2.16864E-02	.05140	5.22618E-02
.15	.02800	2.88492E-02	.07369	7.44222E-02
.20	.03376	3.44408E-02	.09389	9.44998E-02
.25	.03820	3.87213E-02	.11212	1.12616E-01
.30	.04157	4.19381E-02	.12850	1.28897E-01
.35	.04407	4.43137E-02	.14318	1.43463E-01
.40	.04591	4.60396E-02	.15625	1.56434E-01
.45	.04722	4.72744E-02	.16784	1.67924E-01
.50	.04815	4.81451E-02	.17804	1.78039E-01
.55	.04879	4.87509E-02	.18696	1.86877E-01
.60	.04922	4.91671E-02	.19469	1.94531E-01
.65	.04951	4.94495E-02	.20131	2.01084E-01
.70	.04970	4.96390E-02	.20689	2.06609E-01
.75	.04982	4.97644E-02	.21151	2.11172E-01
.80	.04989	4.98462E-02	.21520	2.14828E-01
.85	.04994	4.98983E-02	.21803	2.17624E-01
.90	.04996	4.99298E-02	.22003	2.19596E-01
.95	.04997	4.99466E-02	.22121	2.20768E-01
1.00	.04998	4.99519E-02	.22161	2.21157E-01

Table 23. concluded

```
000000000 310000000 0000000001000000000 000000000 000000000 000000000 0000000000
     INVERSE SOLUTION FOR A PLANE WALL WITH UNIFORM ENERGY GENERATION.
     INITIALLY, T=0. AND ONE FACE IS MAINTAINED AT 0. AND THE OTHER
     IS INSULATED.  THE FCV METHOD IS USED.    TC  IS LOCATED AT X=.5 .
        0.         .3
       .01
       .01        .01         1.
        1.
 0        0.         1.         1.
 1        1.         1.         1.
        0.
    1   20  1.0
0
                           1    4    1    2    2    1    1
        0.         0.     2    3
   11
 0       0.         0.
 0      .01     .01000
 0      .02     .01995
 0      .03     .02969
 0      .04     .03910
 0      .05     .04815
 0      .06     .05683
 0      .07     .06518
 0      .08     .07321
 0      .09     .08096
 0      .10     .08844
 0      .11     .09568
 0      .12     .10269
 0      .13     .10949
 0      .14     .11609
 0      .15     .12251
 0      .16     .12874
 0      .17     .13481
 0      .18     .14072
 0      .19     .14648
 0      .20     .15208
 0      .21     .15754
 0      .22     .16286
 0      .23     .16805
 0      .24     .17311
 0      .25     .17804
 0      .26     .18285
 0      .27     .18754
 0      .28     .19212
 0      .29     .19658
-1      .30     .20093
 0       0.         0.         0.         0.
+1       1.         0.         0.         0.
  1.0        0.0        0.0        0.0        0.0          1
```

Infinite Solid Cylinder with Energy Generation

Consider the infinitely long solid circular cylinder discussed in the second direct sample problem. It is assumed that its surface (x = 0, r = a), the front face, has an energy flux and/or a temperature to be estimated from the history of a single thermocouple measurement located at the midradius, x = 0.5. The initial cylinder temperature is T = 0 and at t = 0, a uniform energy generation rate is started. The thermocouple data are simulated by the exact solution history for x = 0.5. The results of the computations of this inverse heat conduction problem from SODDIT are displayed in Table 24 and are therein compared to its exact solution (cf, Eqs. 5-6, -7).

Solid Sphere with Energy Generation

Consider the solid sphere discussed in the third direct sample problem. It is assumed that its surface (x = 0, r = a), the front face, has an energy flux and/or a temperature to be estimated from the history of a single thermocouple measurement located at the midradius, x = 0.5. The initial temperature of the sphere is T = 0 and at t = 0, a uniform energy generation rate is started. The thermocouple data are simulated by the exact solution history for x = 0.5. The results of the computations of this inverse heat conduction problem from SODDIT are displayed in Table 25 and are therein compared to its exact solution (cf, Eqs. 5-9, 10).

Triangular Heat Flux – Plane Wall

Consider a plane wall with constant material properties. The wall has a width, L = 1 and is insulated at x = L. Initially, the wall was uniformly at T = 0. The intent is to estimate the surface energy flux (at x = 0.), which is known to have a time varying flux: linearly increasing from 0. to 0.6 for $0 \leq t \leq 0.6$ and linearly decreasing from 0.6 to 0. for $0.6 \leq t \leq 1.2$, and zero for $t > 1.2$. This problem is described in detail in Reference 7, pp. 41-49. A single thermocouple is located at x = 1. and its history is simulated using the exact solution for the direct heat conduction problem with the known flux described above. The results for SODDIT are shown in Table 26. Also shown are similar results from program CONTA developed by J. V. Beck and described in Reference 7.

Table 24. Inverse Solution for Uniform Energy Generation Within an Infinitely Long Solid Circular Cylinder. The input data file used by SODDIT is also shown. The SODDIT solution used $\Delta x = .05$, $\Delta t = .01$, fully implicit time integration with the Finite Control Volume method, 2 future times, and 2 time integrations per problem time step.

	t = 0.05		t = 0.25	
	Exact	SODDIT	Exact	SODDIT
$q_s''(t)$	.2260605	-.21364	.4185045	-.41486
Depth	T	T	T	T
.00	.00000	1.8330952E-03	.00000	7.2848417E-04
.05	.01037	1.1740857E-02	.02020	2.0795323E-02
.10	.01898	2.0092447E-02	.03900	3.9494164E-02
.15	.02605	2.6970679E-02	.05642	5.6843292E-02
.20	.03175	3.2514785E-02	.07251	7.2874006E-02
.25	.03629	3.6895527E-02	.08732	8.7625492E-02
.30	.03984	4.0294364E-02	.10090	1.0114151E-01
.35	.04258	4.2887729E-02	.11329	1.1346819E-01
.40	.04465	4.4836687E-02	.12453	1.2465247E-01
.45	.04620	4.6281342E-02	.13468	1.3474108E-01
.50	.04734	4.7338942E-02	.14378	1.4377962E-01
.55	.04816	4.8104523E-02	.15187	1.5181197E-01
.60	.04874	4.8653059E-02	.15899	1.5887967E-01
.65	.04915	4.9042332E-02	.16518	1.6502145E-01
.70	.04943	4.9315954E-02	.17047	1.7027281E-01
.75	.04962	4.9506216E-02	.17490	1.7466573E-01
.80	.04974	4.9636567E-02	.17848	1.7822852E-01
.85	.04982	4.9723667E-02	.18124	1.8098603E-01
.90	.04987	4.9779026E-02	.18320	1.8296091E-01
.95	.04990	4.9810346E-02	.18438	1.8417983E-01
1.00	.04990	4.9823594E-02	.18477	1.8472340E-01

Table 24. concluded

```
000000000 310000000 0000000001000000000 000000000 000000000 000000000 0000000000
     INVERSE SOLUTION FOR INFINITE CYLINDER WITH UNIFORM ENERGY GEN-
     ERATION. INITIALLY, T=0., AND AS TIME PASSES THE OUTER SURFACE IS
     AT 0. .  FCV METHOD USED AND A TC IS LOCATED AT X = .5 .
        0.          .3
       .01
       .01         .01          1.          1.          0.          1.          1.
        1.
 0        0.          1.          1.
 1        1.          1.          1.
        0.
    1   20 1.0
0
                                1    4    1    2    2    1    1
        0.          0.      2    3
   11
 0      0.          0.
 0     .01        .01000
 0     .02        .01993
 0     .03        .02956
 0     .04        .03872
 0     .05        .04734
 0     .06        .05543
 0     .07        .06303
 0     .08        .07016
 0     .09        .07685
 0     .10        .08315
 0     .11        .08907
 0     .12        .09464
 0     .13        .09989
 0     .14        .10483
 0     .15        .10950
 0     .16        .11389
 0     .17        .11804
 0     .18        .12195
 0     .19        .12564
 0     .20        .12912
 0     .21        .13240
 0     .22        .13550
 0     .23        .13842
 0     .24        .14118
 0     .25        .14378
 0     .26        .14624
 0     .27        .14856
 0     .28        .15075
 0     .29        .15281
-1     .30        .15476
 0      0.          0.          0.          0.
+1      1.          0.          0.          0.
  1.0        0.0         0.0         0.0         0.0           1
```

Table 25. Inverse Solution for Uniform Energy Generation Within a Solid Sphere. The input data file used by SODDIT is also shown. The SODDIT solution used $\Delta x = .05$, $\Delta t = .01$, fully implicit time integration with the Finite Control Volume method, 2 future times and 2 time integrations per problem time step.

	t = 0.05		t = 0.25	
	Exact	SODDIT	Exact	SODDIT
$q_s''(t)$	.20231	-.18942	.31615	-.31594
Depth	T	T	T	T
.00	.00000	1.9994757E-03	.00000	1.6339829E-04
.05	.00939	1.0899507E-02	.01535	1.5511338E-02
.10	.01739	1.8605364E-02	.02979	2.9941286E-02
.15	.02412	2.5133283E-02	.04333	4.3465417E-02
.20	.02970	3.0549672E-02	.05598	5.6098911E-02
.25	.03427	3.4957769E-02	.06776	6.7857032E-02
.30	.03795	3.8482304E-02	.07868	7.8754927E-02
.35	.04088	4.1255540E-02	.08875	8.8807953E-02
.40	.04318	4.3406438E-02	.09800	9.8031874E-02
.45	.04495	4.5053381E-02	.10643	1.0644285E-01
.50	.04630	4.6300155E-02	.11406	1.1405726E-01
.55	.04731	4.7234471E-02	.12091	1.2089146E-01
.60	.04806	4.7928249E-02	.12699	1.2696148E-01
.65	.04860	4.8438979E-02	.13233	1.3228281E-01
.70	.04899	4.8811605E-02	.13692	1.3687016E-01
.75	.04927	4.9080552E-02	.14078	1.4073741E-01
.80	.04946	4.9271663E-02	.14393	1.4389772E-01
.85	.04959	4.9403920E-02	.14636	1.4636418E-01
.90	.04967	4.9490931E-02	.14810	1.4815194E-01
.95	.04972	4.9542355E-02	.14914	1.4928611E-01
1.00	.04973	4.9565941E-02	.14948	1.4982056E-01

Table 25. concluded

```
000000000 310000000 0000000001000000000 000000000 000000000 000000000 0000000000
     INVERSE SOLUTION FOR A SOLID SPHERE WITH UNIFORM ENERGY GEN-
     ERATION. INITIALLY, T=0., AND AS TIME PASSES THE OUTER SURFACE IS
     AT 0. .  FCV METHOD USED AND A TC IS LOCATED AT X = .5 .
        0.          .3
       .01
       .01        .01         1.         1.         0.         1.         2.
        1.
 0        0.         1.         1.
 1        1.         1.         1.
        0.
    1   20 1.0
0
                           1    4    1    2    2    1    1
        0.         0.    2    3
   11
 0      0.         0.
 0     .01     .01000
 0     .02     .01990
 0     .03     .02939
 0     .04     .03821
 0     .05     .04630
 0     .06     .05367
 0     .07     .06036
 0     .08     .06643
 0     .09     .07193
 0     .10     .07692
 0     .11     .08144
 0     .12     .08553
 0     .13     .08924
 0     .14     .09260
 0     .15     .09565
 0     .16     .09841
 0     .17     .10090
 0     .18     .10317
 0     .19     .10522
 0     .20     .10708
 0     .21     .10876
 0     .22     .11029
 0     .23     .11167
 0     .24     .11292
 0     .25     .11406
 0     .26     .11509
 0     .27     .11602
 0     .28     .11686
 0     .29     .11763
-1     .30     .11832
 0      0.         0.         0.         0.
+1      1.         0.         0.         0.
  1.0       0.0        0.0        0.0        0.0          1
```

Table 26. Inverse Solution for a Plane Wall Receiving a Triangular Heat Flux. Results are shown for computations from both SODDIT and CONTA[7]. Both programs used identical values of problem constants. The input data file used by SODDIT is also shown. The programs used $\Delta x = .05$, $\Delta t = .03$, semi-implicit ($\theta = .5$) time integration, the Lumped Capacity method, 2 future times, and 2 integrations per problem time steps. Note that CONTA uses the LC method in all but the first and last nodal equation where FCV equations are used. The time, temperature, and flux are dimensionless.

	Surface Temperature		Surface Energy Flux		
Time	CONTA	SODDIT	EXACT	CONTA	SODDIT
-.18	0.	2.84217E-14	0.	0.	-1.1369E-13
-.12	0.	-2.60348E-14	0.	0.	-1.4460E-13
-.06	0.	1.08645E-14	0.	0.	8.7700E-14
.00	.11635E-03	1.18577E-04	0.	.47189E-03	4.7507E-04
.06	.60197E-02	6.13359E-03	0.06	.24179E-01	2.4337E-02
.12	.25709E-01	2.61216E-02	0.12	.91964E-01	9.2357E-02
.18	.50683E-01	5.13186E-02	0.18	.15014E+00	1.5038E-01
.24	.80739E-01	8.16231E-02	0.24	.20956E+00	2.0987E-01
.30	.11550E+00	1.16617E-01	0.30	.27023E+00	2.7053E-01
.36	.15422E+00	1.55536E-01	0.36	.32984E+00	3.3013E-01
.42	.19691E+00	1.98418E-01	0.42	.38997E+00	3.9027E-01
.48	.24335E+00	2.45034E-01	0.48	.44985E+00	4.5015E-01
.54	.29356E+00	2.95392E-01	0.54	.50993E+00	5.1023E-01
.60	.34721E+00	3.49183E-01	0.60	.56902E+00	5.6931E-01
.66	.39294E+00	3.94807E-01	0.54	.58153E+00	5.8151E-01
.72	.41474E+00	4.16117E-01	0.48	.50594E+00	5.0545E-01
.78	.42963E+00	4.30659E-01	0.42	.44966E+00	4.4949E-01
.84	.43797E+00	4.38591E-01	0.36	.39082E+00	3.9050E-01
.90	.44049E+00	4.40729E-01	0.30	.32938E+00	3.2907E-01
.96	.43878E+00	4.38675E-01	0.24	.27030E+00	2.7002E-01
1.02	.43269E+00	4.32253E-01	0.18	.20990E+00	2.0960E-01
1.08	.42274E+00	4.22010E-01	0.12	.15024E+00	1.4995E-01
1.14	.40886E+00	4.07866E-01	0.06	.90039E-01	8.9740E-02
1.20	.39134E+00	3.90111E-01	0.	.30466E-01	3.0173E-02
1.26	.37593E+00	3.74604E-01	0.	-.56932E-02	-5.8307E-03
1.32	.37062E+00	3.69407E-01	0.	.21404E-02	2.2367E-03
1.38	.36688E+00	3.65723E-01	0.	.15150E-03	8.6661E-05

Table 26. concluded

```
000000000 300000000 000000000 000000000 000000000 000000000 000000000 00000000000
      TRIANGULAR HEAT FLUX PROBLEM FOR A PLANE WALL WITH THE
      FACE AT X=L INSULATED.  THE WALL IS INITIALLY AT T=0. EVERYWHERE.
      LUMPED CAPACITY AND 2 FUTURE TIMES.
      -.24      1.38
       .06
       .03       .03        .5
        1.
 0        0.        1.        1.
 1        1.        1.        1.
        0.
    1   20 1.0
0
                          1    4    1    2    2    1    1
                          2    3
   21
 0     -.24        0.
 0     -.18        0.
 0     -.12        0.
 0     -.06        0.
 0       .0        0.
 0      .06    .000007
 0      .12    .000374
 0      .18    .002171
 0      .24    .006323
 0      .30    .013381
 0      .36    .023656
 0      .42    .037319
 0      .48    .054465
 0      .54    .075145
 0      .60    .099389
 0      .66    .1272
 0      .72    .157880
 0      .78    .189293
 0      .84    .219593
 0      .90    .247680
 0      .96    .272931
 0     1.02    .295006
 0     1.08    .313714
 0     1.14    .328954
 0     1.20    .340666
 0     1.26    .348823
 0     1.32    .353762
 0     1.38    .356545
 0     1.44    .358089
-1     1.50    .358941
 0       0.
+1       2.
```

Temperature-Dependent Thermal Properties – Composite Plane Wall

Consider a composite plane wall of total thickness L = 1.05 m shown in Figure 13. The wall is composed of four different materials. Initially, the wall is uniformly at 300K, but at t = 0, a linearly increasing surface flux that is zero at t = 0, but increases to 900,000 W/m^2 at t = 30s, is started at x = 0. At t = 30s, the flux is turned off (i.e., the x = 0 surface is insulated). The face at x = L is always insulated. The complete problem is described in Reference 7, pp. 49-58.

The inverse problem to be solved is to evaluate the surface energy flux and temperature histories using the data from two thermocouples located at x = 0.005 m and 0.01 m. The thermocouple data used here was generated in Reference 7 using a numerical solution to the direct problem. It is noted that all properties are constants except that the thermal conductivity of material number 1 varies linearly between 300 to 1,300 K and 1,300 to 3,000 K as

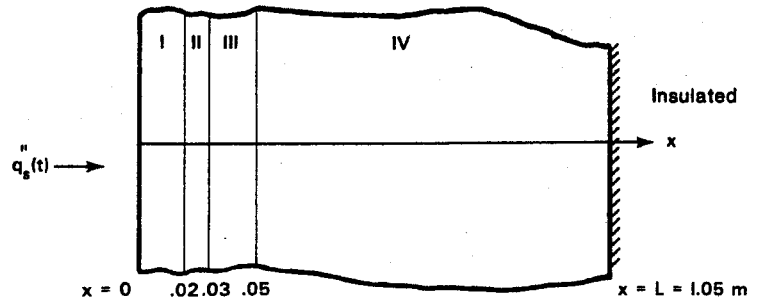
$$k_1 = \begin{cases} 1.0\ \text{W/m-K}, & T \le 300\text{K} \\ \left[\frac{9}{1000}(T-300)+1\right]\text{W/m-K}, & 300 \le T \le 1300\text{K} \\ \left[\frac{10}{1700}(T-1300)+10\right]\text{W/m-K}, & 1300 \le T \le 3000\text{K} \end{cases}$$

The remaining material properties are included in Figure 13.

Table 27 compares the results of SODDIT and program CONTA[7] for the inverse heat conduction solution to this problem. It is noted that CONTA uses a lumped capacitance method for all nodal equations, except the first and last equations where the finite control volume method is used. In addition, CONTA evaluates the thermal conductivity in the first nodal equation at the temperature of node 1, not at the average first-element temperature. The average temperature would be the temperature at the control volume face, the location at which the volume's flux is evaluated. (SODDIT uses a uniformly applied lumped capacity method with k evaluated at the average element temperature.) While the two programs' results do not exactly agree, they are close. Since the input data was generated using CONTA, it is expected that CONTA would produce better results than SODDIT.

Plane Wall with Temperature-Dependent Conductivity and Specific Heat

Consider the plane wall with thermal conductivity and specific heat varying linearly with temperature as described in the eighth direct sample problem. It is the aim of this subsection to present the inverse heat conduction solution for the surface energy flux and temperature for this problem using data from two thermocouples located at x/L = 0.5 and 1.0. The thermocouple history data was taken from the exact solution, Eqs. (5-23, -25, -26). The same input parameters used in the direct problem are used here. The results are summarized in Table 28 where results for the exact, SODDIT, and CONTA solutions are compared.



TC_1 @ x = 0.005m

TC_2 @ x = 0.010m

T(x,0) = 300K

Material I: 3 regions

Δx_1 = .005 m NE_1 = 5

Δx_2 = .005 m NE_2 = 5

Δx_3 = .010 m NE_3 = 10

$$k(T) = \begin{cases} 1.0 \text{ W/m-K @ 300K} \\ \vdots \\ 10.0 \text{ W/m-K @ 1,300K} \\ \vdots \\ 20 \text{ W/m-K @ 3000 K} \end{cases}$$

ρC_p = 3,000,000 J/m^3-K

Material II: 2 regions

$\Delta x_4 = \Delta x_5$ = .005m $NE_4 = NE_5$ = 1

k = 100.0 W/m-K

ρC_p = 4,000,000 J/m^3-K

Material III: 2 regions

Δx_6 = 0.020 m NE_6 = 2

k = 200.0 W/m-K

ρC_p = 4,000,000 J/m^3-K

Material IV: 1 region

Δx_7 = 1.00 m NE_7 = 5

k = 0.010 W/m-K

ρC_p = 10,000,000 J/m^3-K

$$q''_s(t) = \begin{cases} 900{,}000 \frac{W}{m^2} \times \frac{t}{30s} = 30{,}000 \frac{W}{m^2\text{-s}} \, (t[s]) & 0 \le t \le 30s \\ 0 \quad , \quad t > 30s \end{cases}$$

Figure 13. Geometry and Property Values for a Composite Wall with Temperature-Dependent Properties

Table 27. Inverse Solution for a Composite Plane Wall with Temperature-Dependent Properties. Results are shown for computations from both SODDIT and CONTA[7]. Both programs used identical values for problem constants. The input data file used by SODDIT is also shown. Here, two thermocouples (at x = 0.005 m and 0.010 m) were used. Both programs used 3 future times and 4 integrations per problem time step with $\Delta t = 1.0$s and the semi-implicit ($\theta = .5$) time integration procedure. The problem is detailed in Figure 10.

	Surface Temperature, K		Surface Energy Flux, W/m^2		
Time	CONTA	SODDIT	EXACT	CONTA	SODDIT
-6.0	300.00	300.00	0.	4.4754E-05	3.3819E-05
-5.0	300.00	300.00	0.	-6.6289E-05	-4.8798E-05
-4.0	300.00	300.00	0.	-1.2010E-05	7.2598E-06
-3.0	300.00	300.00	0.	7.2600E-05	1.6279E-05
-2.0	300.00	300.00	0.	-2.6446E-05	-1.8698E-05
-1.0	300.00	300.00	0.	-3.4795E-05	2.3868E-05
.0	300.00	300.00	0.	1.9360E-05	6.7155E-06
1.0	308.22	308.3	3. E+04	1.3876E+04	1.6556E+04
2.0	332.79	333.3	6. E+04	4.9203E+04	5.6688E+04
3.0	353.50	354.1	9. E+04	6.7438E+05	7.0149E+04
4.0	400.27	400.6	1.2E+05	1.3655E+05	1.4201E+05
5.0	417.67	418.0	1.5E+05	1.3872E+05	1.2878E+05
6.0	454.15	453.5	1.8E+05	1.8617E+05	1.7849E+05
7.0	487.03	487.9	2.1E+05	2.2275E+05	2.1388E+05
8.0	521.20	521.6	2.4E+05	2.6028E+05	2.4670E+05
9.0	548.07	547.5	2.7E+05	2.7766E+05	2.5901E+05
10.0	581.08	581.0	3.0E+05	3.1335E+05	2.9796E+05
11.0	612.59	613.2	3.3E+05	3.4413E+05	3.2896E+05
12.0	642.50	642.6	3.6E+05	3.6942E+05	3.5109E+05
13.0	671.60	671.3	3.9E+05	3.9305E+05	3.7348E+05
14.0	702.26	702.3	4.2E+05	4.2259E+05	4.0471E+05
15.0	733.31	733.6	4.5E+05	4.5295E+05	4.3539E+05
16.0	764.58	764.7	4.8E+05	4.8332E+05	4.6433E+05
17.0	795.12	795.0	5.1E+05	5.1066E+05	4.9058E+05
18.0	826.53	826.5	5.4E+05	5.4130E+05	5.2155E+05
19.0	858.10	858.2	5.7E+05	5.7199E+05	5.5232E+05
20.0	889.71	889.8	6.0E+05	6.0232E+05	5.8218E+05
21.0	921.33	921.3	6.3E+05	6.3235E+05	6.1172E+05
22.0	953.03	953.0	6.6E+05	6.6250E+05	6.4167E+05
23.0	984.95	985.0	6.9E+05	6.9334E+05	6.7244E+05
24.0	1016.8	1016.8	7.2E+05	7.2360E+05	7.0250E+05
25.0	1048.7	1048.7	7.5E+05	7.5387E+05	7.3257E+05
26.0	1080.6	1080.6	7.8E+05	7.8393E+05	7.6247E+05
27.0	1112.6	1112.6	8.1E+05	8.1448E+05	7.9291E+05
28.0	1144.6	1144.7	8.4E+05	8.4480E+05	8.2311E+05
29.0	1167.2	1167.1	8.7E+05	8.3335E+05	8.1107E+05
30.0	1130.1	1129.5	9.0E+05	5.6813E+05	5.4418E+05
31.0	1058.1	1057.0	.0	2.4603E+05	2.2735E+05
32.0	993.70	992.6	.0	7.1634E+05	6.1212E+04
33.0	948.31	947.7	.0	1.8351E+04	1.4494E+04
34.0	916.38	916.2	.0	9.7792E+04	9.1271E+03
35.0	891.32	891.3	.0	8.1835E+03	8.2498E+03
36.0	870.06	870.1	.0	6.0118E+03	5.9100E+03

Table 27. continued

```
000000000 300000000 000000000 000000000 000000000 000000000 000000000 0000000000
  COMPOSITE WALL RECEIVING A RAMP FLUX (IN TIME) AT X = 0. AND THE FACE
  AT X=L INSULATED.  THE WALL IS INITIALLY AT T=300 K EVERYWHERE.  THE LC
  METHOD IS USED.  THIS PROBLEM IS GIVEN IN SAND83-7134.
       -7.         5.
        1.
        1.         1.         .5
     1000.
 0        0.      3000.        1.
 0      300.      3000.        1.
 0     1300.      3000.       10.
-1     3000.      3000.       20.
     1000.
 0      300.      4000.      100.
-1     3000.      4000.      100.
     1000.
 0      300.      4000.      200.
-1     3000.      4000.      200.
     1000.
 0      300.     10000.       .01
 1     3000.     10000.       .01
      300.
    1    5  0.005
    1    5  0.005
    1   10  0.01
    2    1  0.005
    2    1  0.005
    3    2  0.02
    4    5  1.0
0
                          1    4    2    3    4    1    1
                          2    3
    6   11
 0     -7.       300.       300.
 0     -6.       300.       300.
 0     -5.       300.       300.
 0     -4.       300.       300.
 0     -3.       300.       300.
 0     -2.       300.       300.
 0     -1.       300.       300.
 0      0.       300.       300.
 0      1.       300.       300.
 0      2.       300.       300.
 0      3.     300.01       300.
 0      4.     300.06       300.
 0      5.     300.19       300.
 0      6.     300.51       300.
 0      7.     301.12       300.
 0      8.     302.21       300.
 0      9.     303.99       300.
 0     10.     306.76     300.01
 0     11.     310.84     300.02
 0     12.     316.62     300.04
 0     13.     324.46     300.07
 0     14.     334.68     300.13
 0     15.     347.46     300.23
 0     16.     362.83     300.38
 0     17.     380.65     300.62
 0     18.     400.69     300.97
 0     19.     422.62     301.51
```

Table 27. concluded

0	20.	446.16	302.28
0	21.	471.01	303.40
0	22.	496.94	304.98
0	23.	523.75	307.18
0	24.	551.28	310.21
0	25.	579.42	314.28
0	26.	608.05	319.67
0	27.	637.10	326.62
0	28.	666.49	335.38
0	29.	696.19	346.13
0	30.	726.14	358.98
0	31.	754.27	373.93
0	32.	768.16	390.86
0	33.	769.58	409.41
0	34.	765.72	428.51
0	35.	759.80	446.78
0	36.	753.11	463.17
0	37.	746.20	477.31
0	38.	739.35	489.27
0	39.	732.67	499.28
-1	40.	726.23	507.64
0	-7.		
+1	50.		

Table 28. Inverse Solution for a Plane Wall with Temperature-Dependent Conductivity and Specific Heat. Results are presented from the exact, SODDIT, and CONTA[7] solutions. Computations used 20 equal elements, problem time step of 0.15, semi-implicit ($\theta = 0.5$) time integration, lumped capacity, 2 future times, and 2 integrations per problem time step. The input data file used by SODDIT is also shown. Recall that the front face boundary condition was 3,000 R. To compare the energy fluxes computed by SODDIT and CONTA with the exact solution, the time was shifted to the midpoint of the problem time (since $\theta = 0.5$).

Surface Energy Flux, Btu/hr-ft^2				Surface Temperature, R		
Time, s	Exact	SODDIT	CONTA	Time, s	SODDIT	CONTA
0.05	2686.251	1992.1	2001.7	0.1	2878.6	2850.6
0.15	1550.901	1497.8	1515.1	0.2	2977.5	2962.7
0.25	1200.342	1168.6	1180.2	0.3	2990.3	2981.7
0.35	1007.573	1009.4	1017.0	0.4	3011.2	3005.6
0.45	871.8965	875.51	880.98	0.5	3019.3	3016.3
0.55	763.1646	764.38	768.01	0.6	3021.7	3020.8
0.65	670.8156	671.13	673.71	0.7	3022.2	3022.9
0.75	590.5512	590.51	592.42	0.8	3021.6	3023.6
0.85	520.1825	520.03	521.48	0.9	3020.6	3023.4
0.95	458.2924	458.14	459.25	1.0	3019.3	3022.7
1.05	403.7957	403.64	404.51	1.1	3017.8	3021.7
1.15	355.7890	355.72	356.41	1.2	3016.3	3020.5
1.25	313.4928	313.47	314.02	1.3	3014.8	3019.2
1.35	276.2257	276.28	276.72	1.4	3013.5	3017.8
1.45	243.3898	243.50	243.90	1.5	3012.2	3016.5
1.55	214.4562	214.59	214.84	1.6	3011.0	3015.2
1.65	188.9627	189.09	189.32	1.7	3009.9	3013.9
1.75	166.4997	166.66	166.86	1.8	3008.9	3012.7

Table 28. concluded

```
000000000 300000000 000000000 000000000 000000000 000000000 000000000 0000000000
  INVERSE COMPUTATION OF THERMAL HISTORY OF A PLANE WALL INITIALLY AT
  TØ WITH A SUDDEN STEP IN ONE FACE TEMPERATURE TO TI.  THE BACK
  FACE IS INSULATED.  BOTH K AND C DEPEND ON T AS: KR(1.+B(T-TR)) .
       -.5         2.          3.
        .1         .1
        .1         .1          .5
     112.3                            THIN METAL WALL, L = Ø.1 INCH.
 Ø     500.       .425       .0017
 1    3000.       .858      .00343
      536.
    1   20 .0083333
Ø
                           1    4    2    2    2    1    1
                           2    3
   11   21
 Ø     -.5        536.        536.
 Ø     -.4        536.        536.
 Ø     -.3        536.        536.
 Ø     -.2        536.        536.
 Ø     -.1        536.        536.
 Ø      Ø.        536.        536.
 Ø      .1      939.76      549.19
 Ø      .2     1387.16      729.34
 Ø      .3     1660.60     1016.08
 Ø      .4     1857.93     1293.66
 Ø      .5     2015.05     1533.86
 Ø      .6     2145.82     1736.78
 Ø      .7     2256.84     1908.04
 Ø      .8     2352.03     2053.27
 Ø      .9     2434.11     2177.14
 Ø     1.      2505.16     2283.32
 Ø     1.1     2566.83     2374.73
 Ø     1.2     2620.47     2453.70
 Ø     1.3     2667.22     2522.14
 Ø     1.4     2708.02     2581.58
 Ø     1.5     2743.68     2633.32
 Ø     1.6     2774.88     2678.43
 Ø     1.7     2802.20     2717.82
 Ø     1.8     2826.14     2752.25
 Ø     1.9     2847.14     2782.38
 Ø      2.     2865.56     2808.77
 Ø      3.     2962.45     2946.81
-1      4.     2989.44     2985.05
 Ø     -5.
+1     10.
```

6. Subroutine Descriptions

The following are brief descriptions of the subroutines in the SODDIT code and their functions:

PROGRAM SODDIT. The main program ascertains that all variables are properly initialized, controls the flow of information, calls the appropriate subroutines, and does area and node-drop calculations for ablation problems.

ANG (Automatic Node Generation). This subroutine generates nodal and element data using a constant ratio between the thickness of adjacent elements. The problem domain can be divided into a maximum of eight subregions with each subregion having the properties of a geometric progression.

BC (Boundary Condition). This subroutine modifies the coefficient matrix for the three different types of boundary conditions available. For each time step, the subroutine is called twice; once each for the front- and back-face boundary conditions.

BTHER. This subroutine contains data for the thermodynamic properties of air in the range 0.001-315 atm and 900-10,800°R. Two dimensional linear interpolation is used on either (ln(p),T) or (ln(p),h). This routine is used *only* for Option 1 calculations in which the convective heat transfer coefficient is corrected for how wall effects. This routine is called from CHCORR.

CHCORR (CH CORRection). This subroutine corrects the convective heat transfer coefficient for blowing (ablation) and hot wall effects and is used for Option 1 calculations only. It is called from either SRFBAB OR SRFBNA.

DIDDLE. This subroutine handles decisions for SUBROUTINE ANG.

FUNCTION FK(x). This function is related to SUBROUTINE ANG.

FORMM (FORM Matrix). This subroutine takes the element capacitance, conduction, moving grid, and energy generation contributions and forms the global coefficient matrix; it is used for Option 1 calculations in which ablation takes place.

FORMSM (FORM Symmetric Matrix). This subroutine takes the element capacitance, conduction, and energy generation contributions and forms the global coefficient matrix; it is used for all options in which the coefficient matrix is symmetric.

FZERO. This subroutine finds the zero of a single nonlinear function using an efficient combination of the bisection and secant rule and was obtained from the scientific computing library at Sandia. This routine is called from ANG.

GAP (Heat transfer by convection and radiation across a GAP). This subroutine accounts for energy transfer across a gap of finite width by convection and radiation. For a zero-width gap, a contact conductance is possible. This subroutine is called from SUBROUTINE FORMM for each gap element and at each time step.

FUNCTION GEOM. This function subprogram is a part of SUBROUTINE ANG.

GEOME (GEOMEtry). This subroutine perform the element area and volume calculations for cylindrical and spherical geometries.

GETDT (GET time-step DT). This subroutine chooses the problem time-step DT based on several criteria. 1) the nodal temperature change less than a prescribed amount, 2) having the problem time match a print time, 3) insuring that the diagonal term in the coefficient matrix remains positive, and 4) that DT is between DTMIN and DTMAX. If the time step necessary to match a print time is smaller than DTMIN, then the time step is allowed to be less than DTMIN. It is called from main program for each time step.

GETHC [GET enthalpy (H) of Char material]. This subroutine integrates the tabular heat-capacity data (for material 1) with respect to temperature and adjusts them such that HC is zero at the temperature TREF and is called from SUBROUTINE INPUT.

GETTAB (GET Temperature at which ABlation begins). This subroutine uses input thermochemistry data to determine the minimum temperature at which ablation occurs and is used with Option 1 calculations only.

INPOUT (OUTput of the INPut information). This subroutine prints the majority of the input information and is called from the main program once for each problem.

INPUT (INPUT information). This subroutine reads (nearly) all of the input information and calculates element area and volume; it is called from the main program once for each problem.

INV (INVerse calculations). This subroutine performs (nearly) all of the calculations associated with inverse problems and is called from the main program once for each time step. If the code is to be used only for direct problems, then this subroutine could be deleted.

LOOK (LOOK up in tables). This subroutine is a general purpose one-dimensional linear interpolation subroutine and is called from many places in the code.

LOOK2D (LOOK up in 2D tables). This subroutine is a general purpose two-dimensional linear interpolation subroutine. LOOK2D calls LOOK twice. This subroutine is used only for Type 1 boundary conditions and could be deleted if this option is not used.

OUTPUT (OUTPUT of information at print times). This subroutine prints the entire temperature field and certain energy balance information at selected print times and is called from the main program only at print times. It also fills thermocouple temperature arrays that are subsequently printed in SUBROUTINE SUMRY.

SOURCE (energy SOURCE). This subroutine evaluates the element energy source term. If the user wants a source function that is different from the standard cases already available, then this subroutine must be modified appropriately.

SRFBAB (SuRFace energy Balance ABlation). This subroutine iteratively satisfies the Type 1 boundary condition surface energy balance using surface recession rate as the iteration variable, provided the temperature is above the minimum temperature at which ablation occurs. It is not related to inverse problems.

SRFBNA (SuRFace energy Balance Non-Ablating). This subroutine iteratively satisfies the Type 1 boundary condition surface energy balance for temperatures below which ablation occurs; the iteration variable is surface temperature. It is not related to inverse problems.

SUMRY (SUMmaRY of calculations). This subroutine prints a summary of temperature vs. time for selected depths below the heated surface and is called from the main program once for each problem.

TRIDBS (TRIDiagonal Back Substitution). This subroutine performs the back substitution portion of the Thomas algorithm for a symmetric coefficient matrix and is used for Option 1 calculations only.

TRIDFE (TRIDiagonal Forward Elimination). This subroutine performs only the forward elimination portion of the Thomas algorithm for a non-symmetric coefficient matrix, starting at the back-face node and working toward the front-face node and is used for Option 1 calculations only.

TRIDS (TRIDiagonal Symmetric). Solves the tridiagonal equations for a symmetric coefficient matrix. This routine should not be used for nonsymmetric coefficient matrices.

TRIDSFE (TRIDiagonal Symmetric Forward Elimination). This subroutine performs the forward elimination portion of a symmetric tridiagonal matrix and should not be used for nonsymmetric matricees.

References

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[3]H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, second edition (New York: Oxford University Press, 1959).

[4]E. M. Thorn, "TWIG—A Computer Program for Calculating Unidirectional Transient and Steady State Temperature Distributions in One-, Two-, and Three-Layer Slabs and Cylinders—Constant Heat Flux Applied to Outside Surface," NWEF Report 1038 (Albuquerque, NM: US Naval Weapons Evaluation Facility).

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APPENDIX A

Example of Printed Output for a Direct Heat Conduction Problem

PLANE WALL INITIALLY AT UNIFORM TEMPERATURE
FIXED SURFACE TEMPERATURE AT X = 0 AND INSULATED AT X = L
UNIFORM ENERGY GENERATION

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	0	0	0	0	0	0	0	0	3	1	0	0	0	0	0	0	0	0
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TIMES WHEN PRINT INTERVAL CHANGES/PRINT INTERVALS

0.000E+00 1.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+000.000E+00
1.000E-01 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00

DEPTHS WHERE TEMPERATURE HISTORIES ARE PRINTED

0.000E+00 1.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+000.000E+00

DTMIN	DTMAX	THETA	DTEMPM	SIGMA	RADIUS	EXPN
1.000E-03	1.000E-03	1.000E+00	2.500E+01	4.761E-13	0.000E+00	0.000E+00

MATERIAL PROPERTY DATA

MATERIAL NUMBER 1 :

RHO = 1.000E+00 DHF = 0.000E+00 TREF = 0.000E+00

I	TEMPERATURE	CP	K	EMIT	ENTHALPY
1	0.000E+00	1.000E+00	1.000E+00	0.000E+00	0.000E+00
2	1.000E+00	1.000E+00	1.000E+00	0.000E+00	1.000E+00

THE SOURCE TERM COEFFICIENTS IN MATERIAL NO. 1 ARE:

QA = 1.0000 QB = 0.00000E+00 QC = 0.00000E+00
QD = 0.00000E+00 QE = 0.00000E+00

NODAL DATA

NODE	X	INITIAL TEMP
1	0.00000E+00	0.00000E+00
2	5.00000E-02	0.00000E+00
3	1.00000E-01	0.00000E+00
4	1.50000E-01	0.00000E+00
5	2.00000E-01	0.00000E+00
6	2.50000E-01	0.00000E+00
7	3.00000E-01	0.00000E+00
8	3.50000E-01	0.00000E+00
9	4.00000E-01	0.00000E+00
10	4.50000E-01	0.00000E+00
11	5.00000E-01	0.00000E+00
12	5.50000E-01	0.00000E+00
13	6.00000E-01	0.00000E+00

14	6.50000E-01	0.00000E+00
15	7.00000E-01	0.00000E+00
16	7.50000E-01	0.00000E+00
17	8.00000E-01	0.00000E+00
18	8.50000E-01	0.00000E+00
19	9.00000E-01	0.00000E+00
20	9.50000E-01	0.00000E+00
21	1.00000E+00	0.00000E+00

ELEMENT DATA

NE	NI	NJ	NMAT	DXI	AREAI	VOLI	NGAP
1	1	2	1	5.00000E-02	1.00000E+00	5.00000E-02	0
2	2	3	1	5.00000E-02	1.00000E+00	5.00000E-02	0
3	3	4	1	5.00000E-02	1.00000E+00	5.00000E-02	0
4	4	5	1	5.00000E-02	1.00000E+00	5.00000E-02	0
5	5	6	1	5.00000E-02	1.00000E+00	5.00000E-02	0
6	6	7	1	5.00000E-02	1.00000E+00	5.00000E-02	0
7	7	8	1	5.00000E-02	1.00000E+00	5.00000E-02	0
8	8	9	1	5.00000E-02	1.00000E+00	5.00000E-02	0
9	9	10	1	5.00000E-02	1.00000E+00	5.00000E-02	0
10	10	11	1	5.00000E-02	1.00000E+00	5.00000E-02	0
11	11	12	1	5.00000E-02	1.00000E+00	5.00000E-02	0
12	12	13	1	5.00000E-02	1.00000E+00	5.00000E-02	0
13	13	14	1	5.00000E-02	1.00000E+00	5.00000E-02	0
14	14	15	1	5.00000E-02	1.00000E+00	5.00000E-02	0
15	15	16	1	5.00000E-02	1.00000E+00	5.00000E-02	0
16	16	17	1	5.00000E-02	1.00000E+00	5.00000E-02	0
17	17	18	1	5.00000E-02	1.00000E+00	5.00000E-02	0
18	18	19	1	5.00000E-02	1.00000E+00	5.00000E-02	0
19	19	20	1	5.00000E-02	1.00000E+00	5.00000E-02	0
20	20	21	1	5.00000E-02	1.00000E+00	5.00000E-02	0

TIME DEPENDENT BOUNDARY CONDITION DATA

	IBCTY	IBCTA	AWALL	TRAD	NINVN	NFT	NDTTQ	IFST	ISKP
1	2	1	0.00	0.00	0	1	1	1	1
N	3	2	0.00	0.00					

TIME DEPENDENT BOUNDARY CONDITION TABLE NO. 1

I	TIME	HR	QRAD	CH	P
1	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	1.00000E-20
2	1.00000E+01	0.00000E+00	0.00000E+00	0.00000E+00	1.00000E-20

TIME DEPENDENT BOUNDARY CONDITION TABLE NO. 2

I	TIME	HR	QRAD	CH	P
1	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	1.00000E-20
2	1.00000E+01	0.00000E+00	0.00000E+00	0.00000E+00	1.00000E-20

```
   SANDIA ONE-DIMENSIONAL DIRECT AND INVERSE THERMAL PROGRAM
                  SODDIT VERSION 04-01-86

     PLANE WALL INITIALLY AT UNIFORM TEMPERATURE
     FIXED SURFACE TEMPERATURE AT X = 0 AND INSULATED AT X = L
     UNIFORM ENERGY GENERATION

   THE DATE IS (DD-MMM-YY): 27-AUG-86 ,
   EXECUTION  BEGAN AT (HH.MM.SS) : 09:25:16 AND
   THETA = 1.00  KR(12)=1

  ******************** TIME =  2.00000E-01 ********************

   TIME STEP =  1.00000E-03          NUMBER OF TIME STEPS =  200

        SURFACE ENERGY BALANCE INFORMATION, ITER=  0
   QCOND         QDIFF         QRIN          QROUT         QCHEM         QNET
-4.7933E-01   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00  -4.7933E-01

    BP            MDOTC         SDOT          STOT        PRESSURE        TSG
 0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   5.0394E-21

    HC            HW            HR            CH            CHO         CH/CHO
 0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   1.0000E+00


 NODE MATL MATR     DEPTH        TEMPERATURE   NODE MATL MATR     DEPTH        TEMPERATURE
   1    0    1  0.0000000E+00  5.0393785E-21    12    1    1  5.5000000E-01  1.5896028E-01
   2    1    1  5.0000000E-02  2.3966345E-02    13    1    1  6.0000000E-01  1.6493883E-01
   3    1    1  1.0000000E-01  4.5588173E-02    14    1    1  6.5000000E-01  1.6999347E-01
   4    1    1  1.5000000E-01  6.5019860E-02    15    1    1  7.0000000E-01  1.7420599E-01
   5    1    1  2.0000000E-01  8.2413581E-02    16    1    1  7.5000000E-01  1.7764789E-01
   6    1    1  2.5000000E-01  9.7918253E-02    17    1    1  8.0000000E-01  1.8038013E-01
   7    1    1  3.0000000E-01  1.1167851E-01    18    1    1  8.5000000E-01  1.8245288E-01
   8    1    1  3.5000000E-01  1.2383374E-01    19    1    1  9.0000000E-01  1.8390532E-01
   9    1    1  4.0000000E-01  1.3451719E-01    20    1    1  9.5000000E-01  1.8476553E-01
  10    1    1  4.5000000E-01  1.4385512E-01    21    1    0  1.0000000E+00  1.8505038E-01
  11    1    1  5.0000000E-01  1.5196610E-01
```

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   SANDIA ONE-DIMENSIONAL DIRECT AND INVERSE THERMAL PROGRAM
                    SODDIT VERSION 04-01-86

     PLANE WALL INITIALLY AT UNIFORM TEMPERATURE
     FIXED SURFACE TEMPERATURE AT X = 0 AND INSULATED AT X = L
     UNIFORM ENERGY GENERATION

   THE DATE IS (DD-MMM-YY): 27-AUG-86 ,
   EXECUTION  BEGAN AT (HH.MM.SS) : 09:25:16 AND
   THETA = 1.00  KR(12)=1

  ******************** TIME =  4.00000E-01 ********************

   TIME STEP =  1.00000E-03          NUMBER OF TIME STEPS =  400

        SURFACE ENERGY BALANCE INFORMATION, ITER=  0
   QCOND         QDIFF         QRIN          QROUT         QCHEM         QNET
-6.7290E-01   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00  -6.7290E-01

    BP            MDOTC         SDOT          STOT        PRESSURE        TSG
0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   6.9767E-21

    HC            HW            HR            CH            CHO         CH/CHO
0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   1.0000E+00


 NODE MATL MATR      DEPTH        TEMPERATURE    NODE MATL MATR      DEPTH        TEMPERATURE
   1    0    1  0.0000000E+00  6.9766782E-21      12    1    1  5.5000000E-01  2.5236153E-01
   2    1    1  5.0000000E-02  3.3645045E-02      13    1    1  6.0000000E-01  2.6425396E-01
   3    1    1  1.0000000E-01  6.4883250E-02      14    1    1  6.5000000E-01  2.7460665E-01
   4    1    1  1.5000000E-01  9.3807199E-02      15    1    1  7.0000000E-01  2.8347136E-01
   5    1    1  2.0000000E-01  1.2050833E-01      16    1    1  7.5000000E-01  2.9089357E-01
   6    1    1  2.5000000E-01  1.4507635E-01      17    1    1  8.0000000E-01  2.9691227E-01
   7    1    1  3.0000000E-01  1.6759873E-01      18    1    1  8.5000000E-01  3.0155969E-01
   8    1    1  3.5000000E-01  1.8816011E-01      19    1    1  9.0000000E-01  3.0486109E-01
   9    1    1  4.0000000E-01  2.0684180E-01      20    1    1  9.5000000E-01  3.0683464E-01
  10    1    1  4.5000000E-01  2.2372128E-01      21    1    0  1.0000000E+00  3.0749127E-01
  11    1    1  5.0000000E-01  2.3887172E-01
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   SANDIA ONE-DIMENSIONAL DIRECT AND INVERSE THERMAL PROGRAM
                  SODDIT VERSION 04-01-86

     PLANE WALL INITIALLY AT UNIFORM TEMPERATURE
     FIXED SURFACE TEMPERATURE AT X = 0 AND INSULATED AT X = L
     UNIFORM ENERGY GENERATION

   THE DATE IS (DD-MMM-YY): 27-AUG-86 ,
   EXECUTION  BEGAN AT (HH.MM.SS) : 09:25:16 AND
   THETA = 1.00  KR(12)=1

  ******************** TIME =  6.00000E-01 ********************

   TIME STEP =  1.00000E-03          NUMBER OF TIME STEPS =  600

        SURFACE ENERGY BALANCE INFORMATION, ITER=  0
   QCOND         QDIFF         QRIN          QROUT         QCHEM         QNET
-7.9049E-01   0.0000E+00    0.0000E+00    0.0000E+00    0.0000E+00   -7.9049E-01

    BP            MDOTC         SDOT          STOT         PRESSURE        TSG
 0.0000E+00    0.0000E+00    0.0000E+00    0.0000E+00    0.0000E+00    8.1535E-21

    HC            HW            HR            CH            CHO         CH/CHO
 0.0000E+00    0.0000E+00    0.0000E+00    0.0000E+00    0.0000E+00    1.0000E+00

 NODE MATL MATR     DEPTH        TEMPERATURE   NODE MATL MATR     DEPTH        TEMPERATURE
   1    0    1  0.0000000E+00  8.1534600E-21    12    1    1  5.5000000E-01  3.0933800E-01
   2    1    1  5.0000000E-02  3.9524416E-02    13    1    1  6.0000000E-01  3.2487210E-01
   3    1    1  1.0000000E-01  7.6605710E-02    14    1    1  6.5000000E-01  3.3849270E-01
   4    1    1  1.5000000E-01  1.1130041E-01    15    1    1  7.0000000E-01  3.5023142E-01
   5    1    1  2.0000000E-01  1.4366435E-01    16    1    1  7.5000000E-01  3.6011607E-01
   6    1    1  2.5000000E-01  1.7375231E-01    17    1    1  8.0000000E-01  3.6817048E-01
   7    1    1  3.0000000E-01  2.0161770E-01    18    1    1  8.5000000E-01  3.7441436E-01
   8    1    1  3.5000000E-01  2.2731221E-01    19    1    1  9.0000000E-01  3.7886316E-01
   9    1    1  4.0000000E-01  2.5088551E-01    20    1    1  9.5000000E-01  3.8152797E-01
  10    1    1  4.5000000E-01  2.7238492E-01    21    1    0  1.0000000E+00  3.8241550E-01
  11    1    1  5.0000000E-01  2.9185515E-01
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   SANDIA ONE-DIMENSIONAL DIRECT AND INVERSE THERMAL PROGRAM
                  SODDIT VERSION 04-01-86

     PLANE WALL INITIALLY AT UNIFORM TEMPERATURE
     FIXED SURFACE TEMPERATURE AT X = 0 AND INSULATED AT X = L
     UNIFORM ENERGY GENERATION

   THE DATE IS (DD-MMM-YY): 27-AUG-86 ,
   EXECUTION  BEGAN AT (HH.MM.SS) : 09:25:16 AND
   THETA = 1.00  KR(12)=1

  ********** TIME =  8.00000E-01 **********

   TIME STEP =  1.00000E-03          NUMBER OF TIME STEPS =  800

        SURFACE ENERGY BALANCE INFORMATION, ITER=  0
   QCOND        QDIFF         QRIN         QROUT        QCHEM        QNET
-8.6230E-01   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00  -8.6230E-01

    BP           MDOTC         SDOT         STOT       PRESSURE        TSG
0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   8.8722E-21

    HC           HW           HR           CH           CHO         CH/CHO
0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   1.0000E+00


 NODE MATL MATR     DEPTH        TEMPERATURE   NODE MATL MATR     DEPTH        TEMPERATURE
   1    0    1  0.0000000E+00  8.8721492E-21    12    1    1  5.5000000E-01  3.4413788E-01
   2    1    1  5.0000000E-02  4.3115092E-02    13    1    1  6.0000000E-01  3.6189665E-01
   3    1    1  1.0000000E-01  8.3764926E-02    14    1    1  6.5000000E-01  3.7751365E-01
   4    1    1  1.5000000E-01  1.2198403E-01    15    1    1  7.0000000E-01  3.9100819E-01
   5    1    1  2.0000000E-01  1.5780650E-01    16    1    1  7.5000000E-01  4.0239726E-01
   6    1    1  2.5000000E-01  1.9126580E-01    17    1    1  8.0000000E-01  4.1169542E-01
   7    1    1  3.0000000E-01  2.2239454E-01    18    1    1  8.5000000E-01  4.1891470E-01
   8    1    1  3.5000000E-01  2.5122432E-01    19    1    1  9.0000000E-01  4.2406453E-01
   9    1    1  4.0000000E-01  2.7778545E-01    20    1    1  9.5000000E-01  4.2715171E-01
  10    1    1  4.5000000E-01  3.0210684E-01    21    1    0  1.0000000E+00  4.2818031E-01
  11    1    1  5.0000000E-01  3.2421581E-01
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   SANDIA ONE-DIMENSIONAL DIRECT AND INVERSE THERMAL PROGRAM
                  SODDIT VERSION 04-01-86

     PLANE WALL INITIALLY AT UNIFORM TEMPERATURE
     FIXED SURFACE TEMPERATURE AT X = 0 AND INSULATED AT X = L
     UNIFORM ENERGY GENERATION

   THE DATE IS (DD-MMM-YY): 27-AUG-86 ,
   EXECUTION  BEGAN AT (HH.MM.SS) : 09:25:16 AND
   THETA = 1.00  KR(12)=1

  ******************** TIME =  1.00000E+00 ********************

   TIME STEP =  1.00000E-03          NUMBER OF TIME STEPS = 1000

        SURFACE ENERGY BALANCE INFORMATION, ITER=  0
   QCOND         QDIFF          QRIN          QROUT         QCHEM         QNET
-9.0616E-01   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00  -9.0616E-01

    BP            MDOTC         SDOT          STOT        PRESSURE        TSG
 0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   9.3111E-21

    HC            HW            HR            CH            CHO         CH/CHO
 0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   1.0000E+00


 NODE MATL MATR     DEPTH        TEMPERATURE   NODE MATL MATR     DEPTH        TEMPERATURE
   1    0    1  0.0000000E+00  9.3111179E-21    12    1    1  5.5000000E-01  3.6539336E-01
   2    1    1  5.0000000E-02  4.5308244E-02    13    1    1  6.0000000E-01  3.8451094E-01
   3    1    1  1.0000000E-01  8.8137707E-02    14    1    1  6.5000000E-01  4.0134733E-01
   4    1    1  1.5000000E-01  1.2850948E-01    15    1    1  7.0000000E-01  4.1591432E-01
   5    1    1  2.0000000E-01  1.6644439E-01    16    1    1  7.5000000E-01  4.2822228E-01
   6    1    1  2.5000000E-01  2.0196287E-01    17    1    1  8.0000000E-01  4.3828011E-01
   7    1    1  3.0000000E-01  2.3508485E-01    18    1    1  8.5000000E-01  4.4609516E-01
   8    1    1  3.5000000E-01  2.6582961E-01    19    1    1  9.0000000E-01  4.5167319E-01
   9    1    1  4.0000000E-01  2.9421569E-01    20    1    1  9.5000000E-01  4.5501834E-01
  10    1    1  4.5000000E-01  3.2026074E-01    21    1    0  1.0000000E+00  4.5613311E-01
  11    1    1  5.0000000E-01  3.4398143E-01
```

SANDIA ONE-DIMENSIONAL DIRECT AND INVERSE THERMAL PROGRAM
SODDIT VERSION 04-01-86

PLANE WALL INITIALLY AT UNIFORM TEMPERATURE
FIXED SURFACE TEMPERATURE AT X = 0 AND INSULATED AT X = L
UNIFORM ENERGY GENERATION

THE DATE IS (DD-MMM-YY) :27-AUG-86 ,
EXECUTION BEGAN AT (HH.MM.SS) : 09:25:16 AND
THETA= 1.00 KR(12)=1

**********HEAT FLUX AND TEMPERATURE HISTORIES**********

						DEPTH	0.000E+00	1.000E+00
TIMEQ	QCOND	QDIFF	QRIN	QROUT	QCHEM	TIME	T/C TEMPERATURES	
0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.0000	0.0000
1.000E-01	-3.322E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.000E-01	0.0000	0.0988
2.000E-01	-4.793E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	2.000E-01	0.0000	0.1851
3.000E-01	-5.883E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	3.000E-01	0.0000	0.2537
4.000E-01	-6.729E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	4.000E-01	0.0000	0.3075
5.000E-01	-7.389E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	5.000E-01	0.0000	0.3495
6.000E-01	-7.905E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	6.000E-01	0.0000	0.3824
7.000E-01	-8.308E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	7.000E-01	0.0000	0.4081
8.000E-01	-8.623E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	8.000E-01	0.0000	0.4282
9.000E-01	-8.869E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	9.000E-01	0.0000	0.4439
1.000E+00	-9.062E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.000E+00	0.0000	0.4561

ELAPSED CPU TIME = 0:00:05.69 FOR THIS PROBLEM

APPENDIX B

Example of Printed Output for an Inverse Heat Conduction Problem

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      INVERSE SOLUTION FOR A PLANE WALL WITH UNIFORM ENERGY GENERATION.
      INITIALLY, T=0. AND ONE FACE IS MAINTAINED AT 0. AND THE OTHER
      IS INSULATED.  THE FCV METHOD IS USED.   TC  IS LOCATED AT X=.5 .


      1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20
      0  0  0  0  0  0  0  0  0  0  3  1  0  0  0  0  0  0  0  0
     21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
      0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0
     41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
      0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
     61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
      0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0

         TIMES WHEN PRINT INTERVAL CHANGES/PRINT INTERVALS
0.000E+00 3.000E-01 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+000.000E+00
1.000E-02 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00


         DEPTHS WHERE TEMPERATURE HISTORIES ARE PRINTED
0.000E+00 1.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+000.000E+00


 DTMIN       DTMAX       THETA       DTEMPM      SIGMA       RADIUS       EXPN
1.000E-02  1.000E-02  1.000E+00  2.500E+01  4.761E-13  0.000E+00  0.000E+00


              MATERIAL PROPERTY DATA

MATERIAL NUMBER  1 :

  RHO =  1.000E+00 DHF =  0.000E+00 TREF =  0.000E+00
 I   TEMPERATURE      CP           K           EMIT       ENTHALPY
  1   0.000E+00   1.000E+00   1.000E+00   0.000E+00   0.000E+00
  2   1.000E+00   1.000E+00   1.000E+00   0.000E+00   1.000E+00

THE SOURCE TERM COEFFICIENTS IN MATERIAL NO.   1 ARE:

QA =  1.0000       QB = 0.00000E+00       QC = 0.00000E+00
QD = 0.00000E+00   QE = 0.00000E+00

                 NODAL DATA

        NODE        X          INITIAL TEMP
          1    0.00000E+00    0.00000E+00
          2    5.00000E-02    0.00000E+00
          3    1.00000E-01    0.00000E+00
          4    1.50000E-01    0.00000E+00
          5    2.00000E-01    0.00000E+00
          6    2.50000E-01    0.00000E+00
          7    3.00000E-01    0.00000E+00
          8    3.50000E-01    0.00000E+00
          9    4.00000E-01    0.00000E+00
         10    4.50000E-01    0.00000E+00
         11    5.00000E-01    0.00000E+00
         12    5.50000E-01    0.00000E+00
         13    6.00000E-01    0.00000E+00
```

14	6.50000E-01	0.00000E+00
15	7.00000E-01	0.00000E+00
16	7.50000E-01	0.00000E+00
17	8.00000E-01	0.00000E+00
18	8.50000E-01	0.00000E+00
19	9.00000E-01	0.00000E+00
20	9.50000E-01	0.00000E+00
21	1.00000E+00	0.00000E+00

ELEMENT DATA

NE	NI	NJ	NMAT	DXI	AREAI	VOLI	NGAP
1	1	2	1	5.00000E-02	1.00000E+00	5.00000E-02	0
2	2	3	1	5.00000E-02	1.00000E+00	5.00000E-02	0
3	3	4	1	5.00000E-02	1.00000E+00	5.00000E-02	0
4	4	5	1	5.00000E-02	1.00000E+00	5.00000E-02	0
5	5	6	1	5.00000E-02	1.00000E+00	5.00000E-02	0
6	6	7	1	5.00000E-02	1.00000E+00	5.00000E-02	0
7	7	8	1	5.00000E-02	1.00000E+00	5.00000E-02	0
8	8	9	1	5.00000E-02	1.00000E+00	5.00000E-02	0
9	9	10	1	5.00000E-02	1.00000E+00	5.00000E-02	0
10	10	11	1	5.00000E-02	1.00000E+00	5.00000E-02	0
11	11	12	1	5.00000E-02	1.00000E+00	5.00000E-02	0
12	12	13	1	5.00000E-02	1.00000E+00	5.00000E-02	0
13	13	14	1	5.00000E-02	1.00000E+00	5.00000E-02	0
14	14	15	1	5.00000E-02	1.00000E+00	5.00000E-02	0
15	15	16	1	5.00000E-02	1.00000E+00	5.00000E-02	0
16	16	17	1	5.00000E-02	1.00000E+00	5.00000E-02	0
17	17	18	1	5.00000E-02	1.00000E+00	5.00000E-02	0
18	18	19	1	5.00000E-02	1.00000E+00	5.00000E-02	0
19	19	20	1	5.00000E-02	1.00000E+00	5.00000E-02	0
20	20	21	1	5.00000E-02	1.00000E+00	5.00000E-02	0

TIME DEPENDENT BOUNDARY CONDITION DATA

	IBCTY	IBCTA	AWALL	TRAD	NINVN	NFT	NDTTQ	IFST	ISKP
1	4	1	0.00	0.00	1	2	2	1	1
N	3	2	0.00	0.00					

T/C TABLE NO AND CORRESPONDING NODE NO FOR INVERSE SOLUTION
1 11

TIME DEPENDENT BOUNDARY CONDITION TABLE NO 1
INVERSE OPTION WITH 1 THERMOCOUPLES

I	TIME(I)	Y(I,1)	Y(I,2)	Y(I,3)	Y(I,4)	Y(I,5)	Y(I,6)	Y(I,7)
1	0.000E+00	0.000E+00						
2	1.000E-02	1.000E-02						
3	2.000E-02	1.995E-02						
4	3.000E-02	2.969E-02						
5	4.000E-02	3.910E-02						
6	5.000E-02	4.815E-02						
7	6.000E-02	5.683E-02						
8	7.000E-02	6.518E-02						
9	8.000E-02	7.321E-02						
10	9.000E-02	8.096E-02						
11	1.000E-01	8.844E-02						
12	1.100E-01	9.568E-02						
13	1.200E-01	1.027E-01						
14	1.300E-01	1.095E-01						
15	1.400E-01	1.161E-01						
16	1.500E-01	1.225E-01						
17	1.600E-01	1.287E-01						
18	1.700E-01	1.348E-01						
19	1.800E-01	1.407E-01						
20	1.900E-01	1.465E-01						
21	2.000E-01	1.521E-01						
22	2.100E-01	1.575E-01						
23	2.200E-01	1.629E-01						
24	2.300E-01	1.681E-01						
25	2.400E-01	1.731E-01						
26	2.500E-01	1.780E-01						
27	2.600E-01	1.828E-01						
28	2.700E-01	1.875E-01						
29	2.800E-01	1.921E-01						
30	2.900E-01	1.966E-01						
31	3.000E-01	2.009E-01						

TIME DEPENDENT BOUNDARY CONDITION TABLE NO. 2

I	TIME	HR	QRAD	CH	P
1	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	1.00000E-20
2	1.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	1.00000E-20

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   SANDIA ONE-DIMENSIONAL DIRECT AND INVERSE THERMAL PROGRAM
                  SODDIT VERSION 04-01-86

    INVERSE SOLUTION FOR A PLANE WALL WITH UNIFORM ENERGY GENERATION.
    INITIALLY, T=0. AND ONE FACE IS MAINTAINED AT 0. AND THE OTHER
    IS INSULATED.  THE FCV METHOD IS USED.   TC  IS LOCATED AT X=.5 .

   THE DATE IS (DD-MMM-YY): 27-AUG-86 ,
   EXECUTION  BEGAN AT (HH.MM.SS) : 09:25:16 AND
   THETA = 1.00  KR(12)=1

  ********** TIME =  1.00000E-01 **********

   TIME STEP =  1.00000E-02          NUMBER OF TIME STEPS =   10

        SURFACE ENERGY BALANCE INFORMATION, ITER=  0
    QCOND         QDIFF         QRIN         QROUT         QCHEM         QNET
-3.4791E-01   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00

     BP           MDOTC         SDOT          STOT        PRESSURE        TSG
0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00

     HC           HW            HR            CH           CHO          CH/CHO
0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   1.0000E+00

 NODE MATL MATR     DEPTH        TEMPERATURE   NODE MATL MATR     DEPTH        TEMPERATURE
   1    0    1  0.0000000E+00  1.5827828E-03    12    1    1  5.5000000E-01  9.1025178E-02
   2    1    1  5.0000000E-02  1.7860980E-02    13    1    1  6.0000000E-01  9.3076296E-02
   3    1    1  1.0000000E-01  3.1975202E-02    14    1    1  6.5000000E-01  9.4684483E-02
   4    1    1  1.5000000E-01  4.4081645E-02    15    1    1  7.0000000E-01  9.5929856E-02
   5    1    1  2.0000000E-01  5.4369327E-02    16    1    1  7.5000000E-01  9.6879029E-02
   6    1    1  2.5000000E-01  6.3035394E-02    17    1    1  8.0000000E-01  9.7586140E-02
   7    1    1  3.0000000E-01  7.0273637E-02    18    1    1  8.5000000E-01  9.8093854E-02
   8    1    1  3.5000000E-01  7.6268516E-02    19    1    1  9.0000000E-01  9.8434286E-02
   9    1    1  4.0000000E-01  8.1191798E-02    20    1    1  9.5000000E-01  9.8629779E-02
  10    1    1  4.5000000E-01  8.5200634E-02    21    1    0  1.0000000E+00  9.8693494E-02
  11    1    1  5.0000000E-01  8.8436563E-02
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   SANDIA ONE-DIMENSIONAL DIRECT AND INVERSE THERMAL PROGRAM
                  SODDIT VERSION 04-01-86

    INVERSE SOLUTION FOR A PLANE WALL WITH UNIFORM ENERGY GENERATION.
    INITIALLY, T=0. AND ONE FACE IS MAINTAINED AT 0. AND THE OTHER
    IS INSULATED.  THE FCV METHOD IS USED.   TC  IS LOCATED AT X=.5 .

   THE DATE IS (DD-MMM-YY): 27-AUG-86 ,
   EXECUTION  BEGAN AT (HH.MM.SS) : 09:25:16 AND
   THETA = 1.00  KR(12)=1

  ********** TIME =  2.00000E-01 **********

   TIME STEP =  1.00000E-02          NUMBER OF TIME STEPS =   20

        SURFACE ENERGY BALANCE INFORMATION, ITER=  0
   QCOND        QDIFF         QRIN         QROUT         QCHEM         QNET
-4.9785E-01   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00

    BP           MDOTC         SDOT          STOT        PRESSURE        TSG
 0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00

    HC            HW           HR           CH           CHO        CH/CHO
 0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   1.0000E+00

 NODE MATL MATR     DEPTH        TEMPERATURE   NODE MATL MATR     DEPTH        TEMPERATURE
   1    0    1  0.0000000E+00  1.1883927E-03    12    1    1  5.5000000E-01  1.5899929E-01
   2    1    1  5.0000000E-02  2.4934482E-02    13    1    1  6.0000000E-01  1.6491151E-01
   3    1    1  1.0000000E-01  4.6427495E-02    14    1    1  6.5000000E-01  1.6990798E-01
   4    1    1  1.5000000E-01  6.5763294E-02    15    1    1  7.0000000E-01  1.7407052E-01
   5    1    1  2.0000000E-01  8.3068059E-02    16    1    1  7.5000000E-01  1.7747055E-01
   6    1    1  2.5000000E-01  9.8481776E-02    17    1    1  8.0000000E-01  1.8016886E-01
   7    1    1  3.0000000E-01  1.1214820E-01    18    1    1  8.5000000E-01  1.8221545E-01
   8    1    1  3.5000000E-01  1.2420890E-01    19    1    1  9.0000000E-01  1.8364932E-01
   9    1    1  4.0000000E-01  1.3479994E-01    20    1    1  9.5000000E-01  1.8449845E-01
  10    1    1  4.5000000E-01  1.4405001E-01    21    1    0  1.0000000E+00  1.8477962E-01
  11    1    1  5.0000000E-01  1.5207938E-01
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                  SODDIT VERSION 04-01-86

    INVERSE SOLUTION FOR A PLANE WALL WITH UNIFORM ENERGY GENERATION.
    INITIALLY, T=0. AND ONE FACE IS MAINTAINED AT 0. AND THE OTHER
    IS INSULATED.  THE FCV METHOD IS USED.    TC  IS LOCATED AT X=.5 .

   THE DATE IS (DD-MMM-YY): 27-AUG-86 ,
   EXECUTION  BEGAN AT (HH.MM.SS) : 09:25:16 AND
   THETA = 1.00  KR(12)=1

  ********** TIME =  2.80000E-01 **********

   TIME STEP =  1.00000E-02          NUMBER OF TIME STEPS =   28

        SURFACE ENERGY BALANCE INFORMATION, ITER=  0
   QCOND        QDIFF         QRIN         QROUT         QCHEM         QNET
-5.9277E-01   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00

    BP           MDOTC         SDOT         STOT        PRESSURE        TSG
0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00

    HC           HW           HR           CH           CHO         CH/CHO
0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   1.0000E+00

 NODE MATL MATR     DEPTH        TEMPERATURE    NODE MATL MATR     DEPTH        TEMPERATURE
   1    0    1  0.0000000E+00  9.3088767E-04     12    1    1  5.5000000E-01  2.0200585E-01
   2    1    1  5.0000000E-02  2.9341684E-02     13    1    1  6.0000000E-01  2.1061739E-01
   3    1    1  1.0000000E-01  5.5356273E-02     14    1    1  6.5000000E-01  2.1802759E-01
   4    1    1  1.5000000E-01  7.9094312E-02     15    1    1  7.0000000E-01  2.2430581E-01
   5    1    1  2.0000000E-01  1.0068911E-01     16    1    1  7.5000000E-01  2.2951295E-01
   6    1    1  2.5000000E-01  1.2027133E-01     17    1    1  8.0000000E-01  2.3370118E-01
   7    1    1  3.0000000E-01  1.3796526E-01     18    1    1  8.5000000E-01  2.3691356E-01
   8    1    1  3.5000000E-01  1.5388850E-01     19    1    1  9.0000000E-01  2.3918385E-01
   9    1    1  4.0000000E-01  1.6815233E-01     20    1    1  9.5000000E-01  2.4053629E-01
  10    1    1  4.5000000E-01  1.8086180E-01     21    1    0  1.0000000E+00  2.4098548E-01
  11    1    1  5.0000000E-01  1.9211563E-01
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SANDIA ONE-DIMENSIONAL DIRECT AND INVERSE THERMAL PROGRAM
SODDIT VERSION 04-01-86

INVERSE SOLUTION FOR A PLANE WALL WITH UNIFORM ENERGY GENERATION.
INITIALLY, T=0. AND ONE FACE IS MAINTAINED AT 0. AND THE OTHER
IS INSULATED. THE FCV METHOD IS USED. TC IS LOCATED AT X=.5 .

THE DATE IS (DD-MMM-YY) :27-AUG-86 ,
EXECUTION BEGAN AT (HH.MM.SS) : 09:25:16 AND
THETA= 1.00 KR(12)=1

SUMMARY OF INVERSE CALCULATIONS FOR 1 THERMOCOUPLES

TIME	TSURF	Q	TINPUT	TCALC
0.0100	0.0071	-2.7856E-02	0.0100	0.0100
0.0200	0.0057	-1.2284E-01	0.0199	0.0199
0.0300	0.0016	-2.0352E-01	0.0297	0.0297
0.0400	0.0013	-2.2266E-01	0.0391	0.0391
0.0500	0.0017	-2.4152E-01	0.0482	0.0481
0.0600	0.0023	-2.6053E-01	0.0568	0.0568
0.0700	0.0014	-2.9427E-01	0.0652	0.0652
0.0800	0.0016	-3.1061E-01	0.0732	0.0732
0.0900	0.0014	-3.3242E-01	0.0810	0.0810
0.1000	0.0016	-3.4791E-01	0.0884	0.0884
0.1100	0.0014	-3.6841E-01	0.0957	0.0957
0.1200	0.0014	-3.8421E-01	0.1027	0.1027
0.1300	0.0013	-4.0077E-01	0.1095	0.1095
0.1400	0.0017	-4.1200E-01	0.1161	0.1161
0.1500	0.0008	-4.3695E-01	0.1225	0.1225
0.1600	0.0014	-4.4256E-01	0.1287	0.1287
0.1700	0.0017	-4.5577E-01	0.1348	0.1348
0.1800	0.0017	-4.7070E-01	0.1407	0.1407
0.1900	0.0008	-4.9247E-01	0.1465	0.1465
0.2000	0.0012	-4.9785E-01	0.1521	0.1521
0.2100	0.0014	-5.0910E-01	0.1575	0.1575
0.2200	0.0015	-5.2111E-01	0.1629	0.1629
0.2300	0.0013	-5.3523E-01	0.1681	0.1680
0.2400	0.0010	-5.4893E-01	0.1731	0.1731
0.2500	0.0013	-5.5598E-01	0.1780	0.1780
0.2600	0.0013	-5.6752E-01	0.1828	0.1828
0.2700	0.0015	-5.7604E-01	0.1875	0.1875
0.2800	0.0009	-5.9277E-01	0.1921	0.1921

**********HEAT FLUX AND TEMPERATURE HISTORIES**********

						DEPTH	0.000E+00	1.000E+00
TIMEQ	QCOND	QDIFF	QRIN	QROUT	QCHEM	TIME	T/C TEMPERATURES	
0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.0000	0.0000
1.000E-02	-2.786E-02	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.000E-02	0.0071	0.0100
2.000E-02	-1.228E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	2.000E-02	0.0057	0.0200
3.000E-02	-2.035E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	3.000E-02	0.0016	0.0300
4.000E-02	-2.227E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	4.000E-02	0.0013	0.0400
5.000E-02	-2.415E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	5.000E-02	0.0017	0.0500
6.000E-02	-2.605E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	6.000E-02	0.0023	0.0599
7.000E-02	-2.943E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	7.000E-02	0.0014	0.0697
8.000E-02	-3.106E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	8.000E-02	0.0016	0.0795
9.000E-02	-3.324E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	9.000E-02	0.0014	0.0892
1.000E-01	-3.479E-01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.000E-01	0.0016	0.0987

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1.100E-01 -3.684E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  1.100E-01      0.0014     0.1081
1.200E-01 -3.842E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  1.200E-01      0.0014     0.1173
1.300E-01 -4.008E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  1.300E-01      0.0013     0.1264
1.400E-01 -4.120E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  1.400E-01      0.0017     0.1353
1.500E-01 -4.369E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  1.500E-01      0.0008     0.1440
1.600E-01 -4.426E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  1.600E-01      0.0014     0.1525
1.700E-01 -4.558E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  1.700E-01      0.0017     0.1608
1.800E-01 -4.707E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  1.800E-01      0.0017     0.1690
1.900E-01 -4.925E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  1.900E-01      0.0008     0.1770
2.000E-01 -4.978E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  2.000E-01      0.0012     0.1848
2.100E-01 -5.091E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  2.100E-01      0.0014     0.1924
2.200E-01 -5.211E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  2.200E-01      0.0015     0.1998
2.300E-01 -5.352E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  2.300E-01      0.0013     0.2071
2.400E-01 -5.489E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  2.400E-01      0.0010     0.2142
2.500E-01 -5.560E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  2.500E-01      0.0013     0.2212
2.600E-01 -5.675E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  2.600E-01      0.0013     0.2279
2.700E-01 -5.760E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  2.700E-01      0.0015     0.2345
2.800E-01 -5.928E-01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  2.800E-01      0.0009     0.2410


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